Quantitative Easing and Government Debt Sustainability

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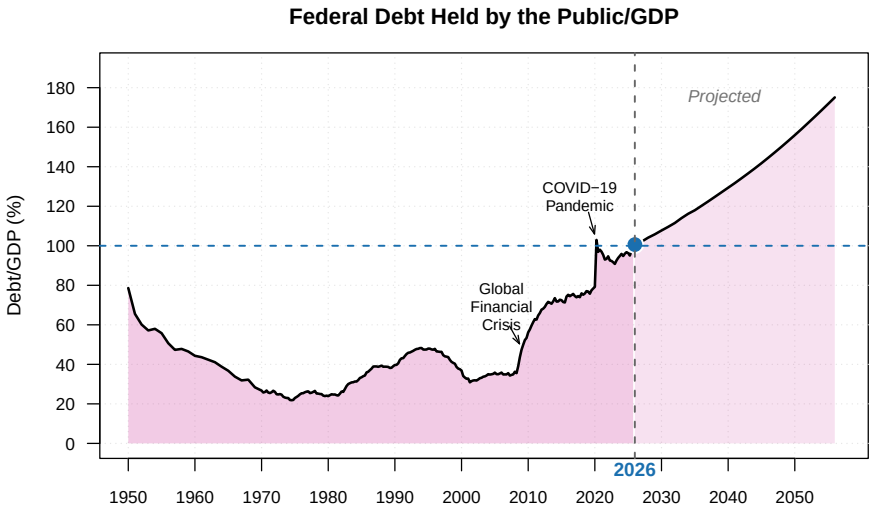
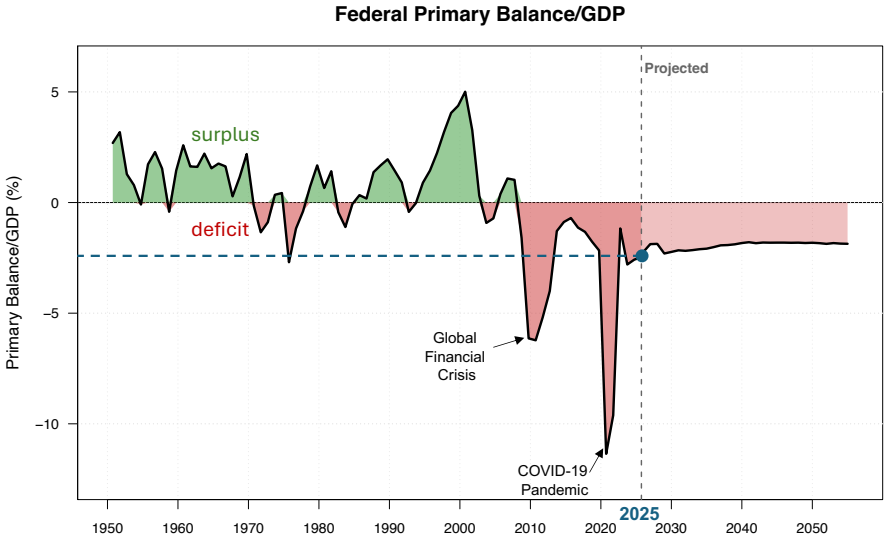
U.S. Debt Sustainability Becomes a Major Policy Concern



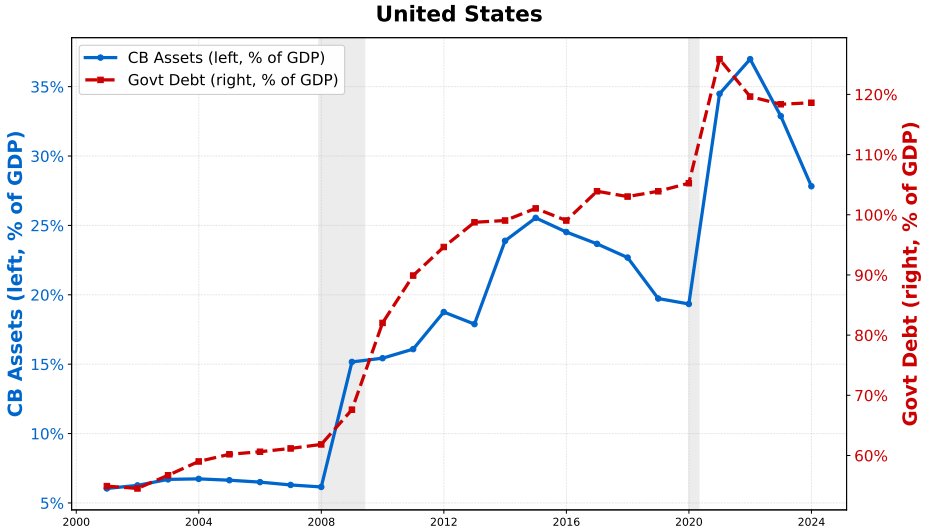
Data: CBO Feb 2026 Long-Term Budget Outlook (history and projections); FRED FYGFGDQ188S.

No Fiscal Adjustment is Projected to Happen So Far



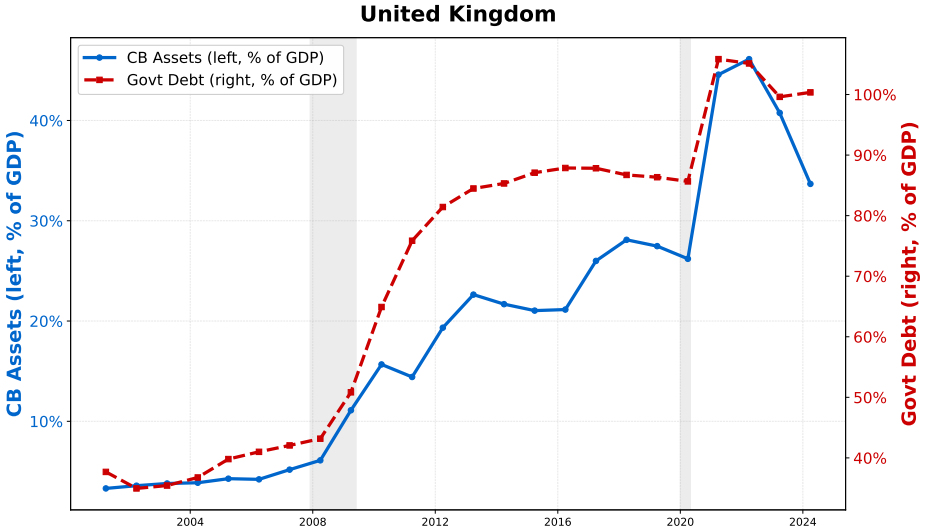
Data Source: Congressional Budget Office.

Federal Reserve Implemented Quantitative Easing as U.S. Govt Debt Expanded



Data: BIS (central bank assets); FRED (government debt, % of GDP).

Bank of England Implemented Quantitative Easing as U.K. Govt Debt Expanded



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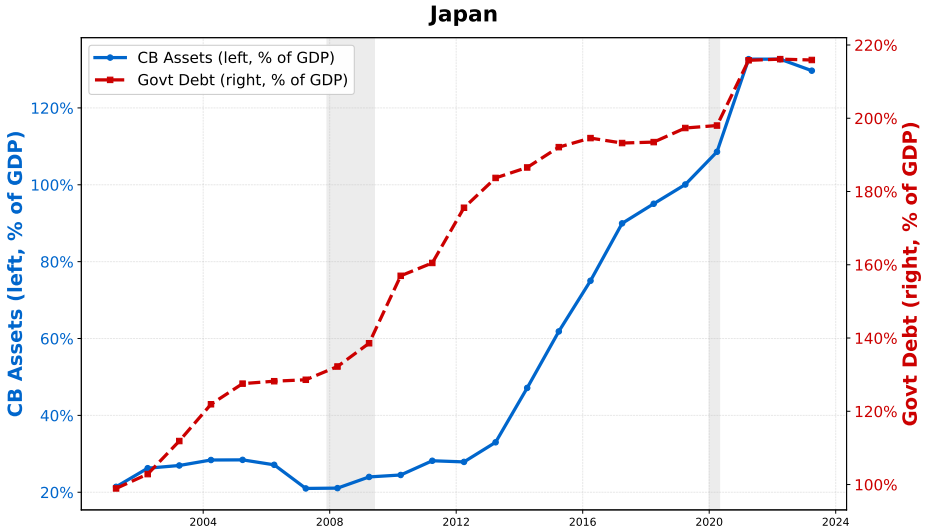
Bank of Japan Implemented Quantitative Easing as Japan's Govt Debt Expanded



Data: BIS (central bank assets); FRED (government debt, % of GDP).

How Do Central Bank Balance-Sheet Expansions Affect Debt Dynamics?

- Canonical view: central bank accommodation eases fiscal pressure and improves debt sustainability.
- Our view: central bank accommodation depletes its long-term capital and reduces debt sustainability.
- Mechanism: QE today $\Rightarrow$ less central bank capital in the future $\Rightarrow$ less future remittances and smaller crisis backstop; debt eventually higher and more fragile.
- Key Results
  - **Structural fragility:** QE increases steady-state debt and lowers default boundary, reducing debt sustainability.
  - **Intervention trap:** Forcing low rates through QE requires a growing CB footprint and accelerating capital depletion.
  - **Sustainability collapse:** Sustainable debt can become unsustainable under very large QE.

The Origin of Central Bank Capital

- **Foreign reserves & real assets**: income-generating external portfolios (FX, gold, other assets) that fund steady remittances.
- **Seigniorage capacity**: revenue from money expansion; aggressive monetization can erode future seigniorage (Laffer-curve logic).
- **Financial repression capacity**: ability to extract resources from banks via reserve requirements, below-market IOR, and captive holdings of public debt; stock-limited and erodes with use.
- **Franchise value / credibility**: institutional capital from independence and reputation that supports stable inflation during crisis support but depletes with repeated extraordinary interventions.

Key feature: **Finite stock and erodes with use**

Literature Review

- Monetary–fiscal interactions: monetary accommodation improves fiscal sustainability

  Sargent and Wallace (1981); Leeper (1991); Sims (1994); Woodford (1995); Cochrane (2023); Bianchi et al. (2023); Bigio et al. (2024); Angeletos et al. (2024); Willems and Zettelmeyer (2022); Reis (2013); Del Negro and Sims (2015).

  - *Our contribution:* due to **finite** CB capital, monetary accommodation worsens fiscal sustainability.

- QE transmission: broad yield compression

  Krishnamurthy and Vissing-Jorgensen (2011); D'Amico and King (2013); Rodnyansky and Darmouni (2017); Di Maggio et al. (2020); Koijen et al. (2021); Selgrad (2023); Greenwood, Hanson, and Stein (2015, 2016); Kiyotaki and Moore (2019); Li (2024); Haddad, Moreira, and Muir (2021, 2024, 2025); Greenwood and Vayanos (2014); Lenel et al. (2017); Vayanos and Vila (2021); Corhay et al. (2023); Ray et al. (2024); Eren et al. (2023); Jansen, Li, and Schmid (2024); Chaudhary et al. (2024); Fang and Xiao (2024).

  - *Our contribution:* yield compression (short-run benefits) $\Rightarrow$ worse debt sustainability (long-run costs).

- Low interest rates ($r < g$) imply fiscal strength

  Blanchard (2019); Mehrotra and Sergeyev (2021); Reis (2021, 2022); Brunnermeier et al. (2020, 2022); Ball and Mankiw (2023); Abel and Panageas (2025); Diamond (1965); Ball, Elmendorf, and Mankiw (1998); Blanchard and Weil (2001).

  - *Our contribution:* low r can be QE-driven and temporary.

Roadmap

1. Model
2. Debt Dynamics without CB
3. Quantitative Easing
4. Conclusion

Environment

- Discrete time and infinite horizon. All variables are in real terms. Government issues one-period real bonds; default indicator $\delta_t \in \{0, 1\}$. Aggregate quantities in upper-case letters.

- Households maximize linear utility plus convenience value $v(\cdot)$:

$$\sum_{t=0}^{\infty} \beta^t \left(c_t + v((1 - \delta_t^h) b_t^h) \right)$$

  - **Vigilante** investors (fraction m): rational belief $\delta_t^v = \delta_t$. No multiplicity: worst-case belief on other vigilantes.
  - **Sticky** investors $(1 - m)$: sluggish belief $\delta_t^s = \delta_{t-1}$.

- We first give a **banking foundation** for central bank capital, then use the generic representation of that same force in the main model.

- Government bond market clearing:

$$\underbrace{B_t}_{\text{total debt}} = \underbrace{B_t^{CB}}_{\text{CB holdings}} + \underbrace{\tilde{B}_t}_{\text{net private holdings}} .$$

Banking Foundation: QE Uses Fiscal Capacity

- Banks fund loans ℓ_t and reserves M_t with captive deposits D:

$$\underbrace{\ell_t}_{\text{loans}} + \underbrace{M_t}_{\text{reserves}} = \underbrace{D}_{\text{deposits}}.$$

  QE raises reserves M_t and crowds out bank lending.

- Lost lending rents and reserve carry reduce the future fiscal surplus:

$$\alpha(M) \equiv \gamma(D - M), \qquad \gamma = \frac{\tau r^{\ell} + (1-\tau) r}{\beta Y} > 0.$$

  Here $\alpha(M)$ is the remaining central-bank/fiscal capacity.

- Therefore QE mechanically lowers central bank capital:

$$\Delta \alpha_{t+1} = -\gamma P_t^L \, \Delta B_{t+1}^{L,CB} < 0.$$

From Banking Foundation to Generic CB Capital

- The banking foundation collapses to the same consolidated budget as the generic model:

$$P_t \tilde{B}_{t+1} + P^Y(\alpha_t - \alpha_{t+1}) = \tilde{B}_t - (\bar{T} + \alpha_t Y).$$

- Interpretation of α_t:
  - **Flow value**: $\alpha_t Y$ improves the consolidated primary balance.
  - **Stock value**: $P^Y \alpha_t$ can be mobilized in a crisis.
- In the banking foundation, $P^Y = 1/(\beta\gamma)$; in the abstract Lucas-tree version, $P^Y = \beta Y/(1-\beta)$.
- From here on, α_t denotes this generic finite CB capital stock.

Government and Generic Central Bank

- Government budget dynamics at time t:

$$\underbrace{P_t^B(B_{t+1} - B_{t+1}^{CB}, \delta_{t+1})B_{t+1}}_{\text{funding supplied by private sector}} + \underbrace{T_t}_{\text{primary surplus}} + \underbrace{T_t^{CB}}_{\text{central bank remittance}} = \underbrace{(1-\delta_t)B_t}_{\text{payment for maturing debt at } t}$$

  - B_{t+1}: issued at time t and due at $t+1$. When multiple solutions, choose smallest one.
  - $P_t^B(\cdot)$: time t government bond price. Depends on next-period default δ_{t+1}.
  - $T_t = \bar{T} < 0$: a constant primary deficit as Sargent and Wallace (1981).

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- Central bank budget dynamics:

$$\underbrace{P^Y \alpha_t}_{\text{old CB capital}} + \underbrace{\alpha_t Y}_{\text{CB capital income}} + \underbrace{(1-\delta_t)B_t^{CB}}_{\text{payment from govt debt}} = \underbrace{P^Y \alpha_{t+1}}_{\text{new CB capital}} + \underbrace{P_t^B B_{t+1}^{CB}}_{\text{new CB debt holding}} + \underbrace{T_t^{CB}}_{\text{CB remittance}}$$

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- Consolidated budget:

$$\underbrace{P^B(\tilde{B}_{t+1}, \delta_{t+1})\tilde{B}_{t+1}}_{\text{funding supply}} + \underbrace{P^Y(\alpha_t - \alpha_{t+1})}_{\text{CB capital rundown}} = \tilde{B}_t - \left(\bar{T} + \alpha_t Y\right)$$

Debt Demand and Pricing

- Debt convenience demand $v(b) = \chi b \left(1 - \frac{1}{2}\frac{b}{b_0}\right)$
- Household bond demand:

$$b^h_{t+1} = (1 - \delta^h_{t+1}) b_0 \max\left\{1 + \chi^{-1}\left(1 - P^B_t/\beta\right), 0\right\}$$

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- Aggregate private-sector demand (assuming no default yet at t):

$$B^h_{t+1} = (1 - m\,\delta_{t+1})\, b_0 \left[1 + \chi^{-1}\left(1 - \frac{P^B_t}{\beta}\right)\right].$$

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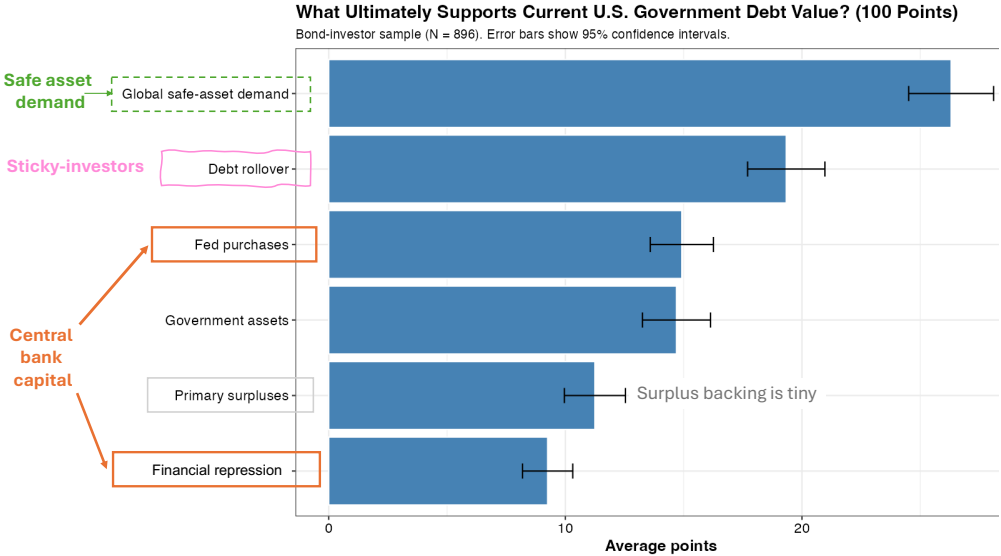
- Price schedule:

$$P^B\left(B_{t+1} - B^{CB}_{t+1},\, \delta_{t+1}\right) = \beta\left[(1+\chi) - \chi\,\frac{B_{t+1} - B^{CB}_{t+1}}{(1 - m\,\delta_{t+1})\, b_0}\right].$$

Government Debt Valuation (survey evidence)



Source: Delao and Li (2026), Beliefs of government debt valuation and sustainability.

Survey Question

Roadmap

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Government Budget (without CB)

- Fiscal authority budget without previous default ($\delta_t = 0$),

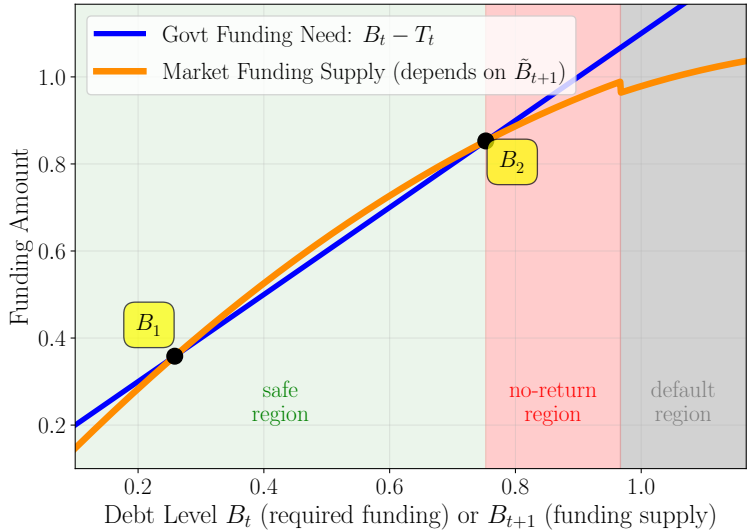
$$\underbrace{P^B\big(B_{t+1},\, \delta_{t+1}\big)\, B_{t+1}}_{\text{market funding supply}} = \underbrace{B_t \;-\; \bar{T}}_{\text{govt funding need}}$$

- Default boundary (no CB):

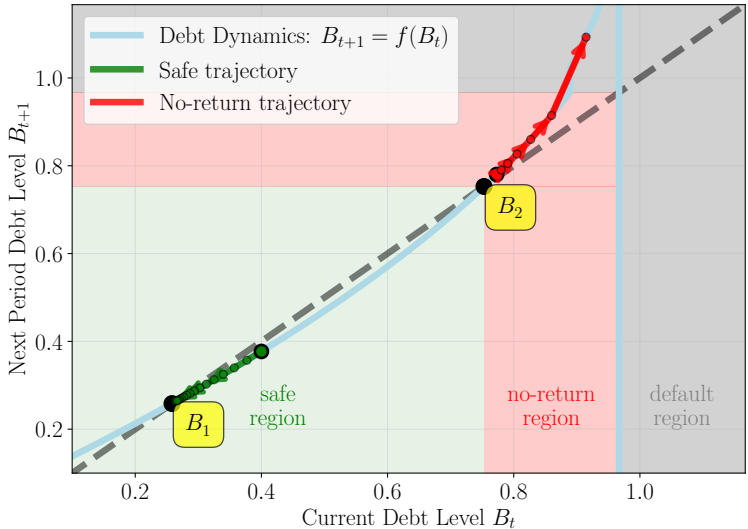
$$\tilde{B}^D = (1-m)\, \max_B \{P^B\big(B, \delta = 1\big) \cdot B\} + \bar{T}.$$

  - Vigilantes assume worse-case scenario: if a default equilibrium is possible, then it is the equilibrium.

Steady States and Sustainability (no CB)



Debt Dynamics (no CB)



Roadmap

1 Model

2 Debt Dynamics without CB

3 Quantitative Easing

4 Conclusion

Consolidating CB and Government Budgets (No default before t)

- Fiscal authority budget:

$$P^B\left(B_{t+1} - B^{CB}_{t+1}, \delta_{t+1}\right) B_{t+1} + \bar{T} + T^{CB}_t = B_t.$$

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- Central bank budget:

$$P^Y_t(\alpha_t - \alpha_{t+1}) + \alpha_t Y + B^{CB}_t = P^B\left(B_{t+1} - B^{CB}_{t+1}, \delta_{t+1}\right) B^{CB}_{t+1} + T^{CB}_t.$$

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- Consolidated budget

$$\underbrace{P^B(B_{t+1} - B^{CB}_{t+1}, \delta_{t+1})\,(B_{t+1} - B^{CB}_{t+1})}_{\text{private-sector funding supply}} + \underbrace{P^Y_t\,(\alpha_t - \alpha_{t+1})}_{\text{CB capital rundown}} = \underbrace{\big(B_t - B^{CB}_t\big)}_{\text{net funding need}} - \underbrace{\big(\bar{T} + \alpha_t Y\big)}_{\text{net primary balance}}.$$

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$$\Rightarrow P^B(\tilde{B}_{t+1}, \delta_{t+1})\tilde{B}_{t+1} + P^Y_t\,(\alpha_t - \alpha_{t+1}) = \tilde{B}_t - \big(\bar{T} + \alpha_t Y\big).$$

Default Boundary with CB at $t+1$

- Default boundary:

$$\tilde{B}^D_{t+1} = (1-m) \max_B \{P^B(B, \delta = 1) \cdot B\} + \bar{T} + \alpha_{t+1}(Y + P^Y)$$

- Vigilantes' belief of government debt default:

$$\delta_{t+1}(\tilde{B}_{t+1}) = \begin{cases} 1, & \tilde{B}_{t+1} > \tilde{B}^D_{t+1} \\ 0, & \tilde{B}_{t+1} \leq \tilde{B}^D_{t+1} \end{cases}$$

- Implications
  - Higher CB capital α_{t+1} raises default boundary $\tilde{B}^D_{t+1}$;
  - QE that depletes CB capital lowers default boundary $\tilde{B}^D_{t+1}$.

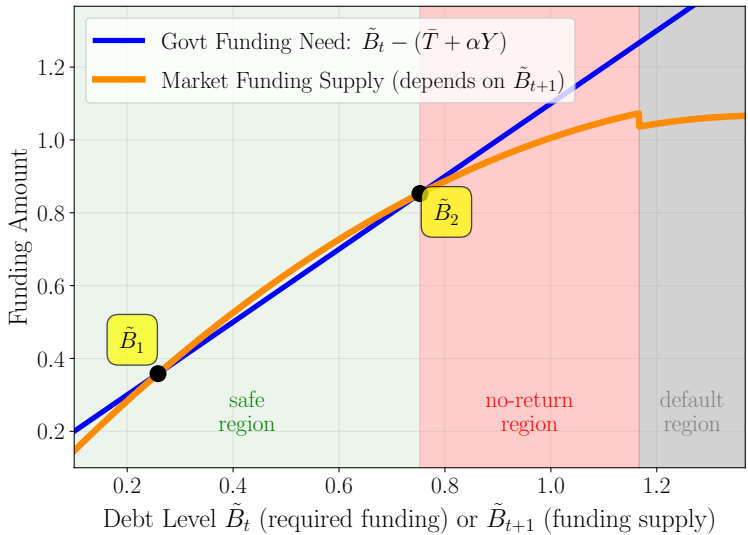
QE: Neutrality Benchmark vs. Reality

- **Benchmark (not in practice)**: If CB keeps capital *constant* and funds purchases by reducing remittances T^{CB} (or equivalently, requiring government transfers), QE is neutral.
  - Net debt dynamics $\tilde{B}_t$ is NOT affected.
  - Total debt $B = \tilde{B}_t + B_t^{CB}$ increases one-for-one with CB purchase.

- **Realistic QE**: CB raises debt holding B^{CB} while **keeping remittances unchanged** $\Rightarrow$ finances purchases by selling capital: $\Delta\alpha < 0$.
  - Implication: today's lower $\tilde{B}$ and lower yields come at the cost of lower future CB capital.

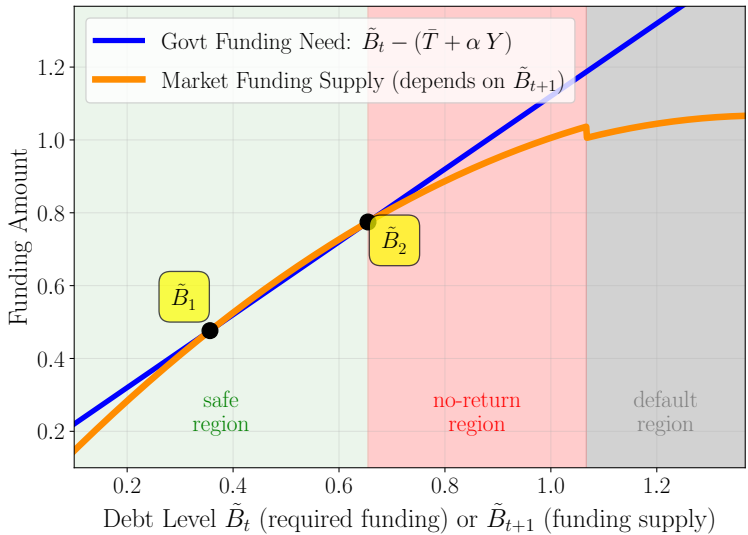
Structural Fragility: QE Worsens Debt Sustainability

Steady States Before QE (CB capital $\alpha > 0$)



Structural Fragility: QE Worsens Debt Sustainability

Steady States After QE (Lower CB capital: $\alpha' < \alpha$)



Structural Fragility: QE Worsens Debt Sustainability

QE policy (increasing B_{t+1}^{CB} while keeping T_t^{CB} fixed) lowers net debt supply and contemporaneous interest rate, but it reduces central bank capital and leads to:

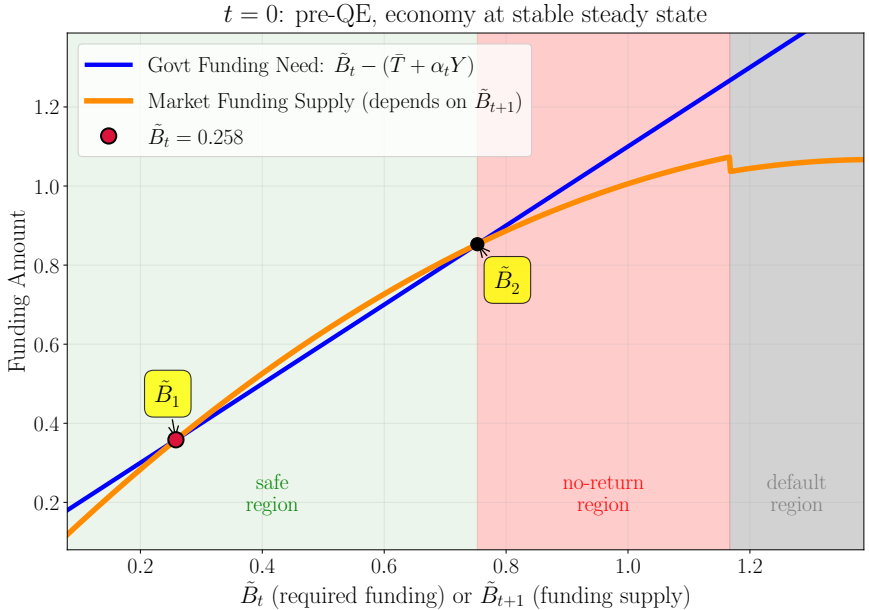
1. higher stable steady-state net debt $\tilde{B}_1^*$.
2. lower unstable steady-state net debt $\tilde{B}_2^*$, so a smaller deficit shock can trigger debt from being sustainable to unsustainable.
3. Lower default boundary.

Central bank accommodation to current fiscal need worsens debt sustainability.

- **Contrast** with monetary-fiscal literature – fiscal accommodation improves debt sustainability

  Sargent and Wallace (1981); Leeper (1991); Sims (1994); Woodford (1995); Cochrane (2023); Bianchi et al. (2023); Bigio et al. (2024); Angeletos et al. (2024); Willems and Zettelmeyer (2022); Reis (2013); Del Negro and Sims (2015).

Intervention Trap: Keeping Rates Low Requires a Growing CB Footprint



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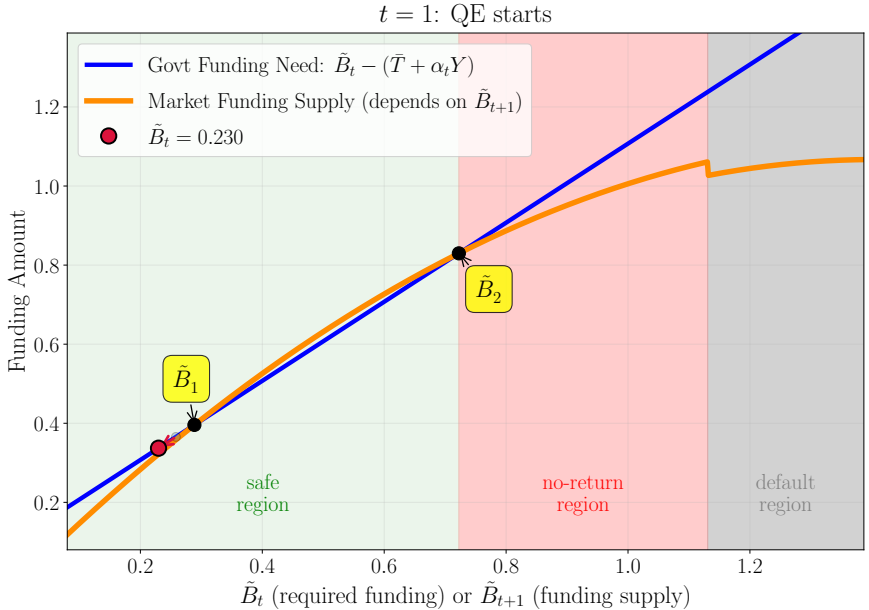

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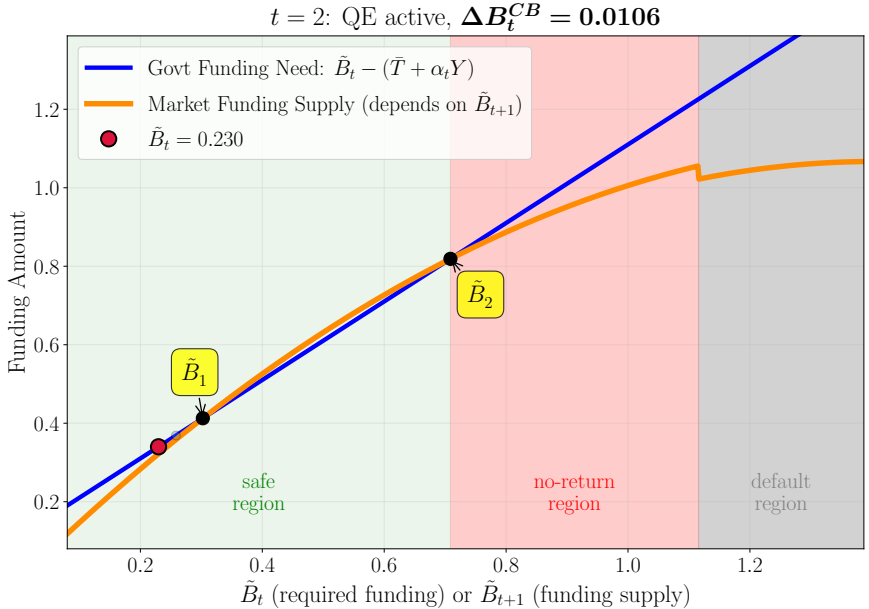

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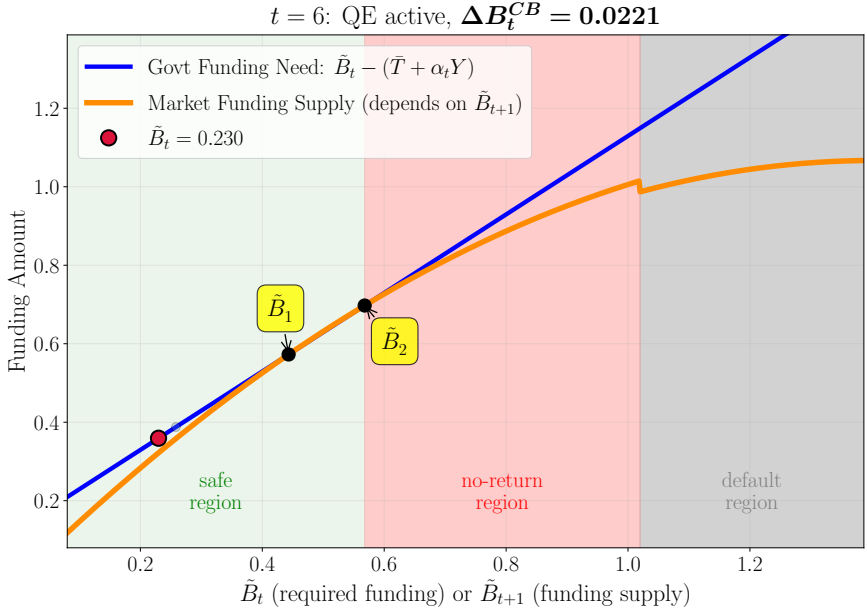

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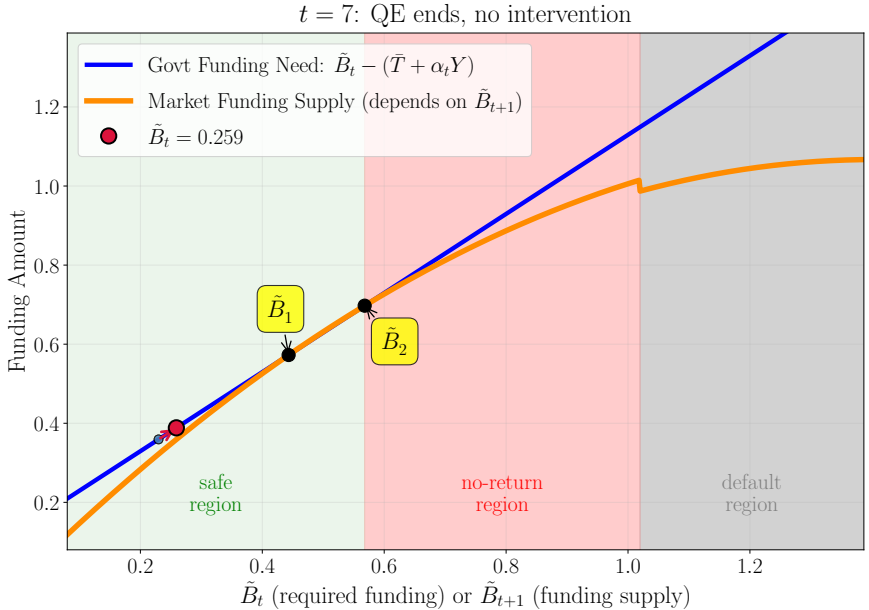

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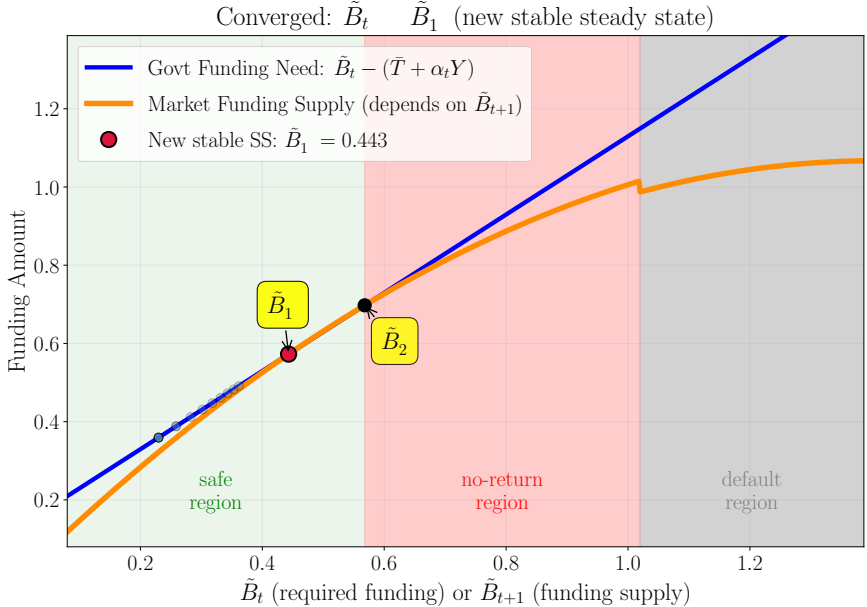
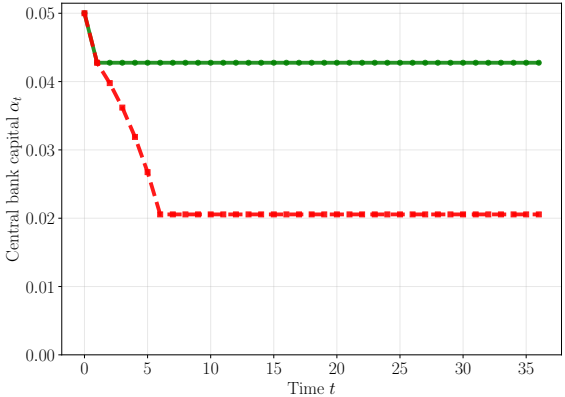
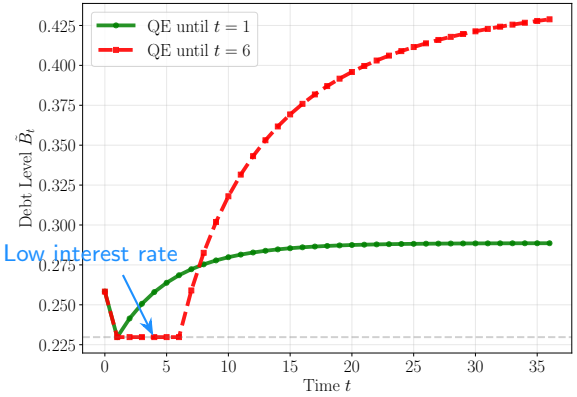


Dynamics of Central Bank Capital and Net Debt under QE



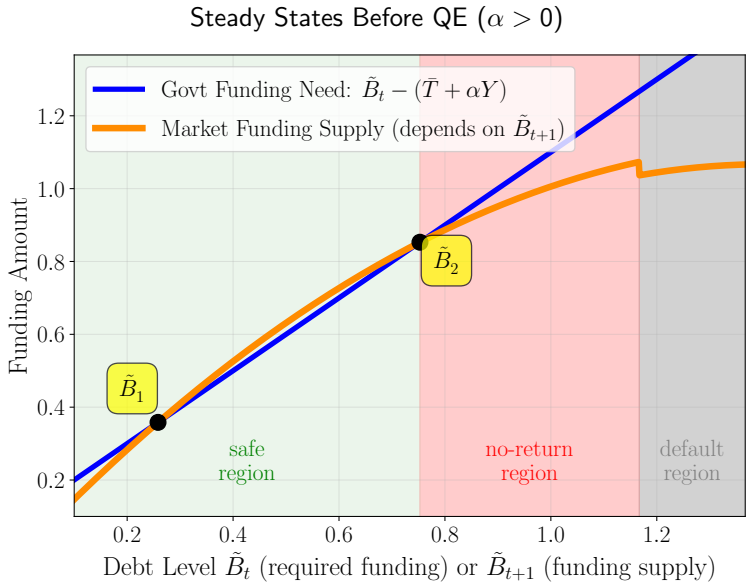
(a) Central bank capital α_t



(b) Net government debt $\tilde{B}_t$

Solid: QE one period; dashed: QE six periods. Both start from stable steady state; QE keeps net debt 10% below initial. Interest path is similar shape as panel (b).

Sustainability Collapse: Large QE Can Induce Explosive Dynamics



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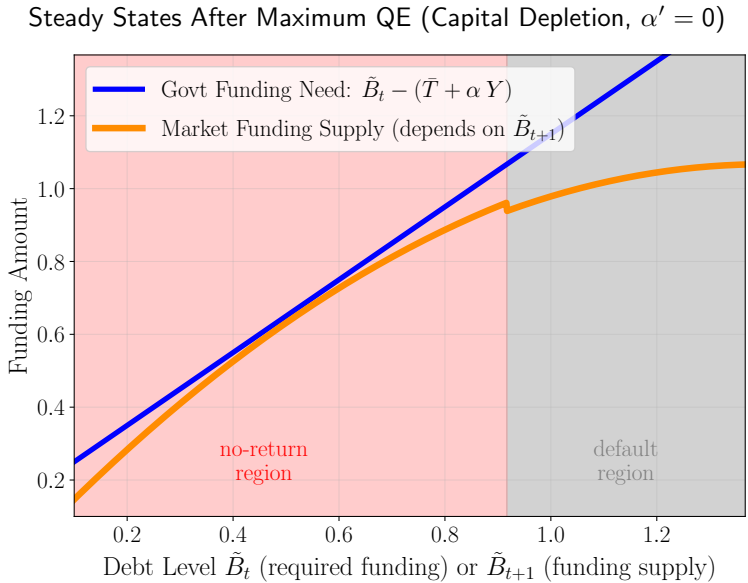
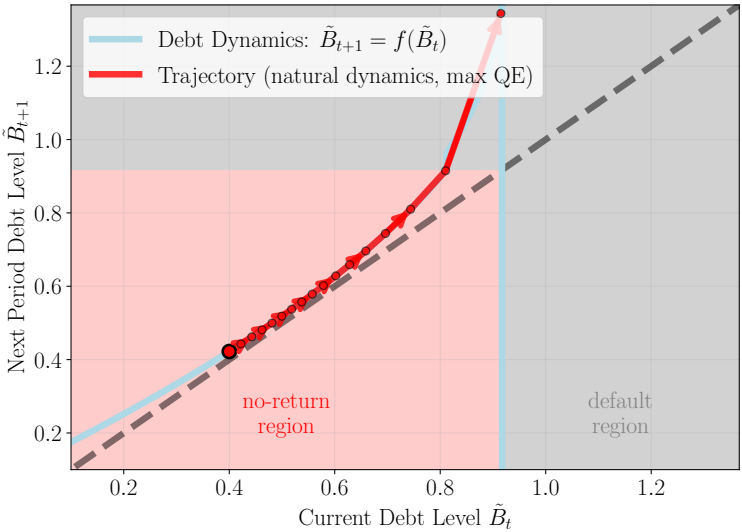


Sustainability Collapse: Large QE Can Induce Explosive Dynamics

Debt Trajectory After Maximum QE (Capital Depletion, $\alpha' = 0$)



Roadmap

1. Model
2. Debt Dynamics without CB
3. Quantitative Easing
4. Conclusion

Conclusion: Short-Run Debt Relief, Long-Run Debt Fragility

- **Structural fragility:** QE draws down finite central-bank resources, worsening debt sustainability.
  (1) higher steady-state debt;
  (2) lower unstable-region boundary;
  (3) lower default boundary.

- **Intervention trap:** Keeping rates low through QE requires a growing CB footprint and accelerating capital depletion, until rates spike and fiscal conditions worsen.

- **Sustainability collapse:** Too much QE can make sustainable debt become unsustainable.

Appendix

Survey Question: Debt Value Decomposition

	Source
(a)	**Primary surpluses:** Future taxes will exceed non-interest spending, allowing the government to service its debt.
(b)	**Debt rollover:** The government will issue new debt to pay off old debt indefinitely.
(c)	**Government assets:** The government has assets (such as land) that can be sold to support the value of its debt.
(d)	**Fed purchases:** The Federal Reserve can buy government debt if needed, supporting its value.
(e)	**Global safe-asset demand:** Investors worldwide value U.S. government debt for its safety and liquidity.
(f)	**Financial repression:** The government will pressure financial institutions to hold large amounts of government debt to support its value.

Source: Delao and Li (2026), Beliefs of government debt valuation and sustainability.

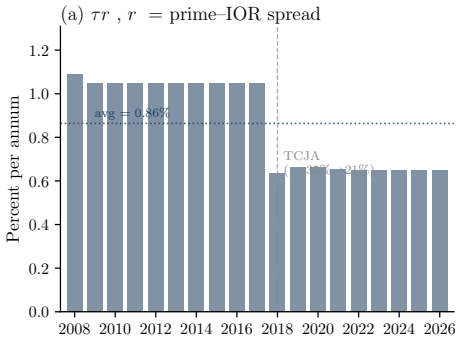
Back

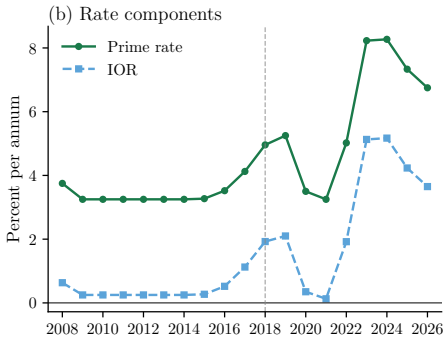
Calibrating the Depletion Coefficient γ

- Banking foundation makes depletion empirically disciplined:

$$d\alpha_{t+1} = -\gamma P_t^L \, dB_{t+1}^{L,CB}, \qquad \gamma > 0.$$

- Two components: lost lending-franchise tax revenue $\tau(r^\ell - r)$ and real carry cost on reserves.
- The lending spread is a persistent feature of bank balance sheets across rate cycles.
- QE looks cheap only when realized real carry is unusually low; the steady-state cost resurfaces when rates normalize.





Banking Foundation: Mapping to α

- QE raises reserves and crowds out loans:

$$\ell_t + M_t = D, \qquad dM_t = P_t^L \, dB_{t+1}^{L,CB}.$$

- Remaining fiscal capacity from the banking franchise:

$$\alpha(M) = \gamma(D - M), \qquad d\alpha_{t+1} = -\gamma P_t^L \, dB_{t+1}^{L,CB} < 0.$$

- This is the concrete foundation for the generic capital stock:

$$P^Y = \frac{1}{\beta\gamma}, \qquad P_t \tilde{B}_{t+1} + P^Y(\alpha_t - \alpha_{t+1}) = \tilde{B}_t - (\bar{T} + \alpha_t Y).$$

Neutrality Benchmark: Purchases Need Financing

- If the CB buys bonds but keeps capital fixed, $\alpha_{t+1} = \alpha_t$, then purchases must be financed through remittances:
$$dT_t^{CB} = -P_t^B(\tilde{B}_{t+1}, \delta_{t+1})\, dB_{t+1}^{CB}.$$
- Consolidation removes the purchase from the real budget:
$$P_t^B(\tilde{B}_{t+1}, \delta_{t+1})\tilde{B}_{t+1} = \tilde{B}_t - (\bar{T} + \alpha_t Y).$$
- Therefore net debt, the default boundary, and sustainability are unchanged.
- Total debt rises one-for-one because
$$B_{t+1} = \tilde{B}_{t+1} + B_{t+1}^{CB}.$$

Theorem 2 Accounting: Holdings vs. Net Purchases

- A low-rate peg is a target for a low net supply, $\tilde{B}' < \tilde{B}_1^*(\alpha_0)$:

$$P^B(\tilde{B}', 0) > P^B(\tilde{B}_1^*(\alpha_0), 0) > 1.$$

- Holding $\tilde{B}'$ fixed requires capital depletion:

$$\Delta\alpha_{s+1} = \frac{g(\tilde{B}') + \bar{T} + \alpha_s Y}{P^Y} < 0, \qquad g(\tilde{B}) = P^B(\tilde{B}, 0)\tilde{B} - \tilde{B}.$$

- As α_s falls, lost capital income makes the drawdown larger:

$$|\Delta\alpha_{s+1}| > |\Delta\alpha_s|.$$

- The **stock** of CB holdings rises because net purchases stay positive:

$$\Delta B_{s+1}^{CB} = \frac{\Delta B_s^{CB}}{P^B(\tilde{B}', 0)} \in (0, \Delta B_s^{CB}), \qquad B_{s+1}^{CB} > B_s^{CB}.$$

Default Boundary: Capacity, Not Balance-Sheet Size

- In the pessimistic refinancing equilibrium, default occurs when net debt exceeds:

$$\tilde{B}_t^D = (1-m)\bar{F} + \bar{T} + \alpha_t(Y + P^Y).$$

- The central bank can support the government with real capacity $\alpha_t(Y + P^Y)$, not with gross bond holdings.
- A QE purchase financed by capital depletion shifts the boundary inward:

$$d\tilde{B}_t^D = (Y + P^Y)d\alpha_t < 0.$$

- Hence QE can lower yields today while making future default possible at lower net debt.