# Public Liquidity and Financial Crises

Wenhao Li*

September 13, 2023

**Abstract**

This paper studies the equilibrium effect of public liquidity on financial crises. Banks borrow from households via insured deposits and partially runnable debt, and suffer endogenous funding withdrawals from households in crises. Holding public liquidity alleviates banks' liquidity problems. In equilibrium, a larger public liquidity supply reduces crisis severity and expands bank lending, but it crowds bank deposits and increases bank vulnerability to real shocks. The model quantitatively explains 40% of Treasury liquidity premium variations. Counterfactual analyses reveal that QE1 significantly improves output, 20 times larger than QE3. However, QE policies raise bank fragility against non-financial shocks such as COVID-19.

*Keywords*: Public liquidity, financial crisis, liquidity premium, banking

*JEL classification*: E44, E58, G01, G28

*Marshall School of Business, University of Southern California. 701 Exposition Blvd, HOH-231, Los Angeles, CA 90089-1422. Phone: 6508232015. Email: liwenhao@marshall.usc.edu. I acknowledge deep gratitude to the mentorship of Arvind Krishnamurthy as my advisor, and the guidance of Darrell Duffie, Sebastian Di Tella, and Yuliy Sannikov on my dissertation committee. I am also grateful to Sam Antill, Juliane Begenau, Shai Bernstein, Laura Blattner, Svetlana Bryzgalova, John H. Cochrane, Itamar Drechsler, Benjamin Hébert, Zhengyang Jiang, Mete Kilic, Pablo Kurlat, Yucheng Liang, Moritz Lenel (discussant), Ye Li (discussant), Hanno Lustig, Yiming Ma, Timothy McQuade, Seth Murray, Sean Myers, Monika Piazzesi, Vincenzo Quadrini, Lukas Schmid, Christopher Tonetti, Jonathan Wallen, Edison G. Yu, Anthony Lee Zhang, and Miao Ben Zhang for helpful discussions, and participants at Stanford Macro lunch, Stanford Finance seminar, Midwest Macroeconomics Meeting, SFS Cavalcade Asia Pacific, NFA, WFA, AFA, HKUST, USC Marshall, and Wharton.

# 1. Introduction

A key lesson from the Great Recession is that a lack of liquidity in the financial sector can be amplified into a fully-fledged financial crisis. To counteract such financial crises, policy-makers generally resort to liquidity regulations and broader emergency liquidity backstops, such as the various liquidity facilities during the Great Recession (Bernanke, 2013). However, liquidity regulations have unintended consequences (Sundaresan and Xiao, 2018), and insuring financial institutions incurs moral hazard problems (Farhi and Tirole, 2012). A third way, which is less studied, is to change the supply of public liquidity (Holmström and Tirole, 1998) and influence the precautionary holding of public liquidity by the financial sector.[1] I will call this channel of influencing banking fragility the "liquidity insurance channel".

Does this channel work in reality, and how important is it? In this paper, I take an equilibrium approach to answer these questions. First, I document that a positive bank leverage shock significantly increases the liquidity premium, consistent with the precautionary liquidity demand from the banking sector. Second, I build a general equilibrium model embedding the liquidity insurance mechanism, where banks suffer from endogenous funding dryups in a crisis, but holding public liquidity alleviates such losses. Third, I show that the model-generated liquidity premium closely tracks the data counterpart, confirming the liquidity insurance channel. Finally, counterfactual analyses reveal the importance of this channel and shed light on the impact of quantitative easing (QE) policies.

My analysis starts from the empirical observation that the liquidity premium positively responds to bank leverage, which is consistent with banks' demand for liquidity. I implement a VAR analysis to extract bank leverage shocks that reflect banks' precautionary liquidity demand. These shocks are orthogonal to contemporaneous changes in confounding factors, such as commercial and industrial loan growth that reflects firm liquidity demand, credit spread that proxies for the credit risk premium, certificate of deposit (CD) rate that reflects bank funding costs, the aggregate stock market return, and many other economic variables. The positive response of the liquidity premium is statistically significant and robust to the ranking of variables and estimation frequency. Furthermore, during events closely related to banking distress, such as the Lehman Brothers bankruptcy and the first release of bank stress testing results, the liquidity premium strongly comoves with bank leverage.

[1] Incorporating financial sector's demand for public liquidity allows the model to speak to financial intermediation and fragility, bank deposit provision, and financial sector health, which is different from the literature that focuses on the the liquidity demand from households and firms (Woodford, 1990; Aiyagari and McGrattan, 1998; Bolton, Chen and Wang, 2011; Guerrieri and Lorenzoni, 2017; Del Negro et al., 2017; Caballero and Farhi, 2018; Liu, Schmid and Yaron, 2019; Herrenbrueck, 2019; Caramp and Singh, 2020). This paper emphasizes QE policy as expanding liquidity supply and thus complements the literature that models QE as purchases of fire-sold assets (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011).

Then I develop a continuous-time general equilibrium model to capture this liquidity insurance channel. The key idea is that banks have a precautionary demand for holding government liquid assets (public liquidity) to guard against aggregate funding withdrawals in future financial crises. The equilibrium value of public liquidity is reflected by the liquidity premium. By supplying more public liquidity, the government can reduce the liquidity premium and expand banks' liquidity holdings, which leads to a more resilient financial system against liquidity shocks, larger bank lending, and higher output. however, it crowds out private liquidity (bank deposits) and increases bank leverage, causing more vulnerability against real shocks on bank asset value.

Public liquidity is modeled as short-term bonds issued by a consolidated government with a given fiscal expenditure policy. The government backs its debt by lump-sum taxation and government holding of private assets, following Gertler and Kiyotaki (2010) and Del Negro et al. (2017). Hereafter, I will use "government bonds" and "public liquidity" interchangeably.[2] An expansion of public liquidity in the model is accompanied by extra government holding of bank debt.

In the model, both banks and households can hold productive capital (interpreted as lending to firms) and public liquidity. Banks have higher productivity in lending; therefore, less bank lending reduces economic output (Brunnermeier and Sannikov, 2014). Banks can raise two types of financing: deposits insured by the government and partially runnable short-term debt. Insured deposits provide additional liquidity value to households, but deposits production is costly for banks. The model has two shocks: an infrequent Poisson shock ("crisis shock") that results in bankruptcy risks and causes household funding withdrawals on the runnable debt, and a Brownian shock to the stock of capital ("capital shock"). In a crisis shock, a small fraction of banks have large exposure to the shock and may go bankrupt. Since households do not know which banks have more exposure, they rationally withdraw their non-deposit lending to banks. As a result, banks need to sell assets to fulfill the funding withdrawals.[3] During these fire sales, the productive capital is subject to liquidation losses, whereas government bonds are not. Thus, when a crisis hits, bank losses are reduced if banks

[2] Mapping to reality, public liquidity refers to the aggregation of bank reserves and Treasury securities. The reason for this consolidation is twofold: (1) both Treasury securities and bank reserves are substitutable for liquidity insurance purposes, and they compose the Level 1 high-quality liquid assets according to post-crisis liquidity regulations; and (2) Treasury securities can be held by all financial institutions to equilibrate the liquidity insurance motive. This aggregation does not capture the "moneyness" of reserves as a transaction device. Such limitation means that the paper will not speak to the effectiveness of exchanging Treasuries and reserves (e.g., typical open market operations and Treasury purchase component of QE).

[3] An alternative setup is that banks do not sell assets but raise costly external financing. This emergency financing cost can be alleviated by posting government bonds as collateral. With appropriate parameters, this alternative setup can generate similar quantitative results. In other words, the model mechanism does not hinge on actual fire sales.

hold more government bonds and liquidate them rather than productive capital.

In equilibrium, due to banks' demand for liquidity, government bonds earn lower yields, as reflected by a positive liquidity premium. Households passively demand a fixed fraction of the total government bond supply. Interpreted as the marginal value of liquidity, the liquidity premium increases with the aggregate demand for liquidity from the financial sector and decreases with higher government bond supply. Therefore, both bank wealth share and the total amount of government bonds are state variables that determine the liquidity premium.

I show that an expansion of public liquidity reduces banks' vulnerability against future financial crises, thus boosting bank lending and economic output in normal times. As a side effect, it crowds out private liquidity provision by bank deposits and raises bank leverage, which increases bank vulnerability to capital shocks.

After dissecting the theoretical mechanism, I map the model to the U.S. economy and calibrate model parameters. The liquidity insurance mechanism generates the liquidity premium as an equilibrium asset price determined by bank wealth and total public liquidity supply. To test this mechanism, I construct the liquidity premium as the principal component of the standard measures in Longstaff (2004) and Nagel (2016). With the bank equity ratio and public liquidity supply as inputs, the model generates equilibrium liquidity premium that accounts for about 40% of the data variations. I find that the nonlinearity in the liquidity insurance mechanism is vital to the quantitative success – in a linear regression, bank equity ratio and public liquidity supply only explain 30% of the variations in the liquidity premium, while using leverage and public liquidity supply boosts $R^2$ to about 40%. The model captures this nonlinearity effect of bank capital. Furthermore, the model-implied impulse response of the liquidity premium to bank leverage shocks is similar to the VAR counterpart.

Next, I use the calibrated model to study the impact of liquidity supply on economic output. With nonlinear endogenous jumps with significant financial amplification, the model successfully matches the economic dynamics around the 2008 financial crisis, including bank equity, the liquidity premium, total consumption, and economic output. Counterfactual analyses reveal that expanded public liquidity supply because of QE1 in 2009 significantly reduced output drop, but QE3 in later years has a much smaller effect, which is mainly due to better bank balance sheet conditions and a larger existing stock of public liquidity. Note that these exercises do not reflect the full impact of QE, since the model is designed only to capture the liquidity insurance effect. A comprehensive evaluation requires the inclusion of other channels, such as the credit constraint channel in Gertler and Kiyotaki (2010).

Finally, I use the model to shed light on the impact of QE policies when the underlying shock is non-financial, such as COVID-19. It is arguably more of a supply-side shock than a direct shock to the financial system. Nevertheless, the Fed still implemented QE policies.

On March 15, 2020, the Fed announced the purchase of $500 billion worth of Treasuries and $200 billion worth of MBS. Then on March 23, 2020, the Fed expanded the QE to "QE infinity", an initiative to purchase an "unlimited" amount of Treasuries and MBS. To illustrate how the channel in this paper works in such an event, I focus on two aspects of the COVID-19 recession – sudden supply-side shocks and a large public liquidity expansion. Surprisingly, counterfactual analyses reveal that QE infinity reduces output. The reason is that the underlying shock is a capital shock, not a bank liquidity shock. The increased supply of public liquidity expands bank lending but also leads to higher fragility against capital shocks, making banks worse off when capital shocks hit.

In summary, this paper contributes to the literature by qualitatively and quantitatively analyzing the liquidity insurance channel that connects the liquidity premium to financial crises. The model builds on the idea that losses in financial crises are attributed to liquidity problems triggered by insolvency concerns. The high explanatory power of the model in explaining the liquidity premium supports that banks are marginal in pricing liquidity. The model reveals that the elasticity of output to public liquidity crucially depends on the amount of public liquidity and the nature of the underlying shock.

**Related Literature.** This paper explores the demand of public liquidity from banks, which plays an important role in allocating credit and providing private liquidity to households. Previous literature focuses on the liquidity substitution effect (Krishnamurthy and Vissing-Jorgensen, 2015; Li, 2017; Quadrini, 2017), and analyzes the role of public liquidity for normal-times (Lenel, 2017; Bianchi and Bigio, 2018; Drechsler, Savov and Schnabl, 2018; Infante and Vardoulakis, 2018; Lenel, Piazzesi and Schneider, 2019; Infante, 2020). In both Drechsler, Savov and Schnabl (2018) and d'Avernas and Vandeweyer (2021), banks are subject to liquidity shocks which can be alleviated by holding public liquidity. Nevertheless, these liquidity shocks do not cause financial crises, since d'Avernas and Vandeweyer (2021) have idiosyncratic shocks while in Drechsler, Savov and Schnabl (2018) banks are fully liquidity insured. My contribution in comparison is to show how shocks to bank net worth affect the liquidity premium and use this connection to quantify the dynamics of financial crises.

My study on banks' liquidity insurance motive complements the literature on firm and household liquidity demand (Woodford, 1990; Aiyagari and McGrattan, 1998; Eisfeldt, 2007; Bolton, Chen and Wang, 2011; Eisfeldt and Muir, 2016; Guerrieri and Lorenzoni, 2017; Del Negro et al., 2017; Caballero and Farhi, 2018; Liu, Schmid and Yaron, 2019; Herrenbrueck, 2019; Caramp and Singh, 2020). The unique features of banks' liquidity demand include its propagation to the macroeconomics via a nonlinear amplification mechanism, and the tight link between bank equity and the liquidity premium. My model features aggregate risks and complements the recent literature that focuses on the insurance against idiosyncratic risks (Di Tella, 2020; Brunnermeier, Merkel and Sannikov, 2020, 2022). As shown by Siriwardane, Sunderam and Wallen (2021), the post-crisis financial markets are segmented, so idiosyncratic risks become more relevant in the post-crisis world. Integrating both systematic and idiosyncratic risks is an interesting direction for further research. For example, Barattieri, Moretti and Quadrini (2021) show that insuring against idiosyncratic risks can create more exposure to systematic risks.

This paper is tightly related to the empirical literature on the liquidity premium of U.S. Treasury securities (Longstaff, 2004; Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood, Hanson and Stein, 2015; Nagel, 2016; Van Binsbergen, Diamond and Grotteria, 2018; Jiang, Krishnamurthy and Lustig, 2018; Fleckenstein and Longstaff, 2020). Nominal interest rate is strongly correlated with the liquidity premium (Nagel, 2016; Drechsler, Savov and Schnabl, 2017). As discussed in Nagel (2016), the reason for the strong relationship between nominal interest rate and the liquidity premium is because of the Fed's active adjustment over effective Treasury supply to the private sector, which is consistent with the view that Treasury supply still affects the liquidity premium. Furthermore, according to Li, Ma and Zhao (2019), d'Avernas and Vandeweyer (2021), and Li and Krishnamurthy (2022), after controlling for the federal funds rate, Treasury supply significantly affects the liquidity premium and bank liquidity provision. This paper focuses on the Treasury supply effect.

This paper also adds a new perspective on how quantitative easing policies work. The existing literature shows that government purchase of private assets can alleviate financial constraints (Gertler and Kiyotaki, 2010; Gertler, Kiyotaki and Queralto, 2012; Benigno and Robatto, 2019). This paper shows that the liquidity creation due to an expanded government balance sheet is also powerful in stimulating the economy and increasing the resilience against financial shocks (but not real capital shocks).

As a technical contribution, this paper enriches the types of continuous-time models in the macro-finance literature[4], by introducing two state variables with endogenous jump sizes in equilibrium. My model thus helps broaden the scope of the literature to encompass richer dynamics with sudden and large disruptions, which are central to financial crises (Schularick and Taylor, 2012; Muir, 2017; Krishnamurthy and Li, 2020).

The modeling of financial crises as a severe liquidity shortage follows a large literature (Holmström and Tirole, 1998; Gorton and Ordonez, 2014; Moreira and Savov, 2017; Gertler and Kiyotaki, 2015). The aggregate funding withdrawal in my model is related to models of

[4]Relevant papers include He and Krishnamurthy (2013, 2014), Brunnermeier and Sannikov (2014), Bianchi and Bigio (2018), Di Tella (2017), Moreira and Savov (2017), Drechsler, Savov and Schnabl (2018), etc. See Hansen, Khorrami and Tourre (2018) for a summary of these models and how to solve them numerically.

financial panics (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005; He and Xiong, 2012; Liu, 2016; Gertler, Kiyotaki and Prestipino, 2020; Liu, Wang and Yang, 2019; Robatto, 2019; Wei and Yue, 2020). My model avoids the typical sunspot problem by introducing real shocks and information asymmetry.

The rest of the paper is organized as follows: Section 2 describes the empirical motivation. Section 3 presents the full model. Section 4 solves the model and describes the theoretical properties of the model. Section 4 provides numerical algorithms and the calibration of the model. Section 5 describes asset pricing implications. Section 6 discusses macroeconomic implications. Section 7 concludes.

# 2. Empirical Evidence

According to the liquidity insurance channel, negative shocks to intermediary risk-bearing capacity increase the liquidity premium. Nevertheless, direct evidence is scant in the literature. This section is devoted to credibly identifying such an effect.

## *2.1. VAR Evidence*

To study the relationship formally, I implement a VAR analysis that extracts orthogonal variations in bank leverage. I discover that the response of the liquidity premium to bank leverage is persistently positive and significant, and robust to various twists. These results suggest a precautionary motive of banks in holding public liquidity.

In particular, the VAR includes variables in the following order: (1) growth of GDP ($\Delta \text{GDP}_t$), which proxies the general demand of liquidity; (2) C&I loan growth ($\Delta \text{C\&I loans}_t$) and (3) investment growth ($\Delta \text{investment}_t$), which both reflect the corporate sector demand for liquidity; (4) The supply of public liquidity (public liquidity/GDP), measured as the ratio of public debt minus Fed holding plus bank reserves over GDP, which captures the idea of public liquidity as liquid assets supplied by the broadly defined government to the private sector; (4) Inflation; (5) Federal funds rate; (6) The aggregate stock market return, which helps alleviate concerns that the correlation between bank leverage and the liquidity premium is driven by a common risk premium; (7) Bank leverage, which is the market leverage of primary dealers as in He, Kelly and Manela (2017); (8) The excess bond premium (data from Gilchrist and Zakrajšek (2012)), which measures the common risk premium component in corporate bond credit spreads; (9) The liquidity premium, measured as the principal component of measures from Longstaff (2004) and Nagel (2016).

To construct the liquidity premium, I use various maturity-matched Refcorp bonds –

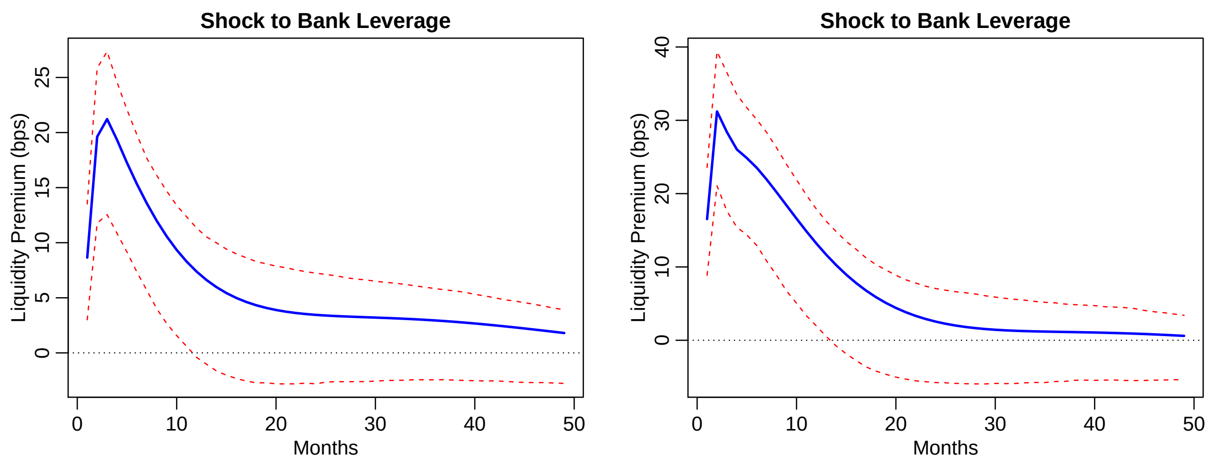
Treasury spreads (1 to 5 years) and the spread between 3-month repos collateralized by Treasuries and 3-month Treasuries from 1991 to 2016. The first measure is used by Longstaff (2004), and the second by Nagel (2016). Guaranteed by the government, Refcorp bonds have the same credit quality as Treasuries. Furthermore, Refcorp bonds receive the same tax treatment as Treasuries (Longstaff, 2004). Therefore, the yield difference between Refcorp bonds and Treasuries comes from their liquidity differentials, which stem mainly from market microstructure differences. The repo–Treasury spread is also a clean measure of the liquidity premium because the term repos collateralized by Treasuries have the same credit quality as the U.S. government. In both measures, the maturities are matched. This principal component approach extracts the most robust feature of various liquidity premium measures. Before 1991, due to data limitation, I follow Nagel (2016) to use the spread between 3-month banker's acceptance and 3-month Treasury bills.[5]

The identification strategy of the above VAR follows the literature by recursively ordering the variables as above, so that the shock to a variable is orthogonalized against all previous variables. In other words, I identify the shock to bank leverage as orthogonal to contemporaneous changes in GDP growth, C&I loan growth, federal funds rate, and other included variables ranked before bank leverage. As a result, by construction, the shock to bank leverage is not driven by other variables. The ordering of variables follows the general practice in the literature, such as putting inflation after GDP and government debt, and interest rate after both inflation and GDP (due to the endogenous policy setting).

In Figure 1, I show the impulse responses of the liquidity premium to a one standard deviation shock of bank leverage, using a VAR setup of two lags. Under the baseline specification, as shown in panel (a), the response in the liquidity premium is positive and significant at around 20 bps, and the significant response persists for about one year. A formal statistical test reveals that the first-year impulse response is significantly positive with 99% confidence ($p$-value below 1%). To the extent that C&I loan growth, investment growth, and GDP growth can proxy liquidity demand from other sources, this significant impulse response to bank leverage indicates a link between bank fragility and the liquidity premium and supports the liquidity insurance channel. In panel (b), I further show that ranking bank leverage as the first variable in the VAR yields similar results.

In the appendix, I examine the robustness of the impulse response. First, I show results using different number of lags. Second, I restrict the sample to non-crises and non-recession

[5]Historically, the banker's acceptance market was actively supported by the central bank and had negligible credit risks due to the double guarantee of both firms and banks. Compared to other possible measures before 1991, such as the federal funds rate–Treasury spread, the banker acceptance spread is more stable and on average lower, indicating fewer credit risk concerns. Refer to the online appendix for further details.




(a) baseline variable ranking

(b) ranking leverage first

Figure 1. **Impulse Responses of the Liquidity Premium to One Standard Deviation Shocks of Bank Leverage.** Dotted red lines illustrate the 90% confidence interval via bootstrapping. The VAR setting for panel (a) is in equation (A42) and (A43), with two lags ($p = 2$). The setting for panel (b) is the same as (a) except that it ranks the bank leverage as the first variable in the VAR. The data sample is from 1973 (constrained by the GZ excess bond premium) to 2016.

periods. I find that the magnitude of the impulse response is smaller, but still highly significant. Third, eliminating a set of variables typically makes the impulse response even stronger. Finally, results are robust to the ranking of variables, such as changing the ranking of the liquidity premium.

In summary, the VAR analysis reveals a positive and robust relationship between bank leverage and the liquidity premium. Given that the impulse responses are orthogonalized against proxies for household and corporate liquidity demand and other macroeconomic variables, these results support the liquidity insurance channel and motivate a formal model to examine its quantitative relevance.

## *2.2. Residualized Bank Leverage and Difference Regressions*

One concern of the VAR results is that bank leverage moves with the overall market, so the results may just pick up the relationship between the liquidity premium and risk premia in general. To address the concern, I residualize the bank leverage measure against systematic risk factors and then implement difference regressions of the liquidity premium on the residualized bank leverage.

Results are shown in Table 1. All regressions include controls for the changes in public liquidity supply and market volatility. In column (1), I show the baseline regression using raw

Table 1: Liquidity Premium and Bank Leverage

| | Dependent Variable: 100∗diff(Liquidity Premium) | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| diff(Lvg) | $2.28^{***}$ | | | | | $2.16^{***}$ |
| | (0.48) | | | | | (0.48) |
| market return | | $-0.48^{***}$ | | | | |
| | | (0.15) | | | | |
| diff(log(Lvg)) | | | $40.55^{***}$ | | | |
| | | | (10.56) | | | |
| diff(Residualized Lvg 1) | | | | $1.94^{***}$ | | |
| | | | | (0.71) | | |
| diff(Residualized Lvg 2) | | | | | $1.53^{**}$ | |
| | | | | | (0.76) | |
| (market return)*Lvg | | | | | | $-0.02^{**}$ |
| | | | | | | (0.01) |
| Observations | 503 | 503 | 503 | 503 | 503 | 503 |
| Adjusted $R^2$ | 0.06 | 0.04 | 0.05 | 0.03 | 0.03 | 0.07 |

*Notes*: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$. The data sample is from 1970 to 2016 at monthly frequency. Liquidity Premium is the principal component of measures from Longstaff (2004) and Nagel (2016) as described in Section 2.1. The unit of the liquidity premium is in percentage points, and in the regression the dependent variable is 100*diff(liquidity premium). "diff" is the monthly difference of variables. Lvg is the bank leverage measure from He, Kelly and Manela (2017). "market return" is the monthly return of S&P 500 index. "diff(Residualized Lvg 1)" is constructed as the residual of diff(Lvg) regressed on the S&P 500 index returns. "diff(Residualized Lvg 2)" is constructed in a similar way, but the regressors include all three Fama-French factors' returns. Control variables include VIX index (extended to 1970 using realized volatility) and public liquidity/GDP as constructed in Section 2.1.

differences of bank leverage and the liquidity premium. In column (2), I show that, indeed, the market return is an important confounding factor. In column (3), I use the difference of log leverage to show that the relation between leverage and the liquidity premium is robust. In column (4), I construct residualized leverage changes as the regression residuals of monthly bank leverage changes on monthly market returns. In column (5), residualized leverage changes are constructed in a similar way, but regressors include all Fama-French three factors. Therefore, columns (4) and (5) present results that are less prone to systematic fluctuations in bank leverage. In column (6), I use the interaction between bank leverage and market return to incorporate the nonlinear effect of market return when bank leverage is high. Across all settings, coefficients on leverage are significant. The magnitudes of coefficients in columns (2) and (3) are slightly smaller than that of column (1), but the differences are not statistically significant. In summary, results indicate that the strong connection between the liquidity premium and bank leverage is not driven by systematic market fluctuations.

### *2.3. Event Studies*

To further support the liquidity insurance mechanism, I also examine the relation between bank leverage and the liquidity premium on event days with significant disruptions in the banking sector. I find that bank leverage and the liquidity premium strongly comove around those events. Details are shown in Appendix C.3.

## 3. Model

In this section, I will present the setup of the model. To start, I fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with the usual conditions and assume that all stochastic processes are adapted. The economy evolves in continuous time. A complete list of notations is provided in Appendix A.2.

### *3.1. Agents and Assets*

The model economy is populated by a continuum of optimizing households and bankers and a government, interpreted as the consolidation of fiscal and monetary authorities. There are four types of traded assets: productive capital, insured deposits, non-deposits, and government bonds (or "public liquidity" as in Holmström and Tirole (1998)).

The household objective is to maximize the standard log utility,

$$E[\int_0^\infty e^{-\rho t} \log(c^h_{j,t}) dt] \tag{1}$$

where $c^h_{j,t}$ is household consumption. Bankers are interpreted as bank equity holders. The objective function for banker $j$ is also to maximize the same log utility subject to the same discount rate $\rho$.

Firms are modeled as productive capital, held either by banks or households. To achieve a growth rate $\mu^K dt$ in capital $k_t$, a flow $\phi(\mu^K_t)k_t dt$ of consumption goods is needed. I assume that $\phi(\mu^K)$ satisfies the following usual conditions:

$$\lim_{\mu^K \to \underline{\mu}^K} \phi'(\mu^K) = 0, \quad \lim_{\mu^K \to \infty} \phi'(\mu^K) = \infty, \quad \phi''(\mu^K) > 0, \tag{2}$$

which represent a typical cost function in the literature that includes both direct investment costs and also adjustment costs. Denote the value of per-unit productive capital as $q_t$. Each firm's objective is to maximize the growth of productive capital, subject to investment adjustment costs. The first-order condition implies

$$q_t = \phi'(\mu_t^K). \tag{3}$$

which is the standard $q$-theory of investment in continuous time.

Households and bankers are compensated for their different levels of skills proportionally (see similar assumptions in Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), and Rampini and Viswanathan (2019)). Banker-operated capital has productivity $\bar{A}$, which is higher than the productivity of household-operated capital, denoted as $\underline{A}$. The difference captures the downside of the economy when banks are constrained. The dynamic evolution of productive capital owned by agent $j$ (either a banker or a household) is

$$\frac{dk_{j,t}}{k_{j,t}} = \mu_t^K dt - \delta dt + \sigma^K dZ_t - \tilde{\kappa}_{j,t} dN_t, \tag{4}$$

where $\delta$ is the depreciation rate, and $\sigma^K$ is capital growth volatility. There are two types of aggregate shocks: capital shocks driven by the Brownian motion $dZ_t$,[6] and crisis shocks driven by the Poisson process $dN_t$. The individual exposure $\tilde{\kappa}_{j,t}$ is independent and has the same distribution $P(\tilde{\kappa}_{j,t} = 1) = \theta$, and $P(\tilde{\kappa}_{j,t} = 0) = 1 - \theta$, among both households and bankers.

When a bank is subject to full exposure, i.e., $\kappa_{j,t} = 1$, all the productive capital held by the bank is destroyed. Since a banker in the model will have infinitely negative utility when the wealth becomes zero, to allow for bankruptcy, I assume that each banker exits with a small fraction $\varepsilon$ of their pre-crisis wealth. In the event of bank capital destruction, $\kappa_{j,t} = 1$, the non-depositors that have already withdrawn funding obtain full payment before the government steps in. Then the government liquidates the bank and makes full payments to insured deposits so that depositors do not suffer any losses.[7] If the asset value is not enough, the government makes up the payment to depositors. If the asset value is above what is needed to cover the insured depositors, the positive value is seized by the government, so the remaining non-depositors get zero out of a liquidated bank.[8] The crisis shock $dN_t$ is mainly a financial shock as I will take $\theta$ close to zero, so disruptions are mostly driven by

[6]Although the $dZ_t$ shock is not needed for the model to generate crises, it is essential for quantitatively matching the output dynamics in non-crises episodes.

[7]These assumptions mimic the actual liquidation process for FDIC-insured banks.

[8]It is not important for the main mechanism whether non-depositors get zero or partial recovery, but the zero-recovery assumption significantly simplifies the model.

the tightened financial constraints instead of capital destruction.

The information about individual bank's exposure $\tilde{\kappa}_{j,t}$ is private at the moment of the aggregate shock $dN_t$. Bank creditors decide whether or not to withdraw their funding from each bank before knowing the actual exposure.[9] Insured deposits are fully guaranteed by the government and thus depositors have no incentive to run on the bank. On the other hand, households suffer a fraction $\theta > 0$ of losses on average over their non-deposits that are not withdrawn. If households do not withdraw the non-deposits, with probability $1 - \theta$, they earn deposit interest rate at an order of $(1 - \theta)dt$, but the expected losses are at the order $\theta \cdot 1$. Therefore, withdrawing all non-deposits is the dominant strategy for all households. To reflect the reality that a fraction of non-deposits are long-term bonds and certain retail investors are sticky, I assume that each household can withdraw up to a fraction $\beta \in (0, 1)$ of non-deposits.[10]

I assume that banks cannot issue outside equity, which is a standard assumption in the literature[11]. Furthermore, in the event of funding withdrawals, banks cannot immediately raise additional insured deposits. One motivation is that deposits are sticky and deposit flows are slow-moving. Another is that even if commercial banks can quickly raise more deposit funding, at the bank holding company level, the broker-dealer branch that suffers funding withdrawals cannot use the deposit financing due to regulatory restrictions. Both reflect the reality that liquidity problems do happen such as the 2008 financial crisis, despite banks' ability to raise funding via insured deposits.

To fulfill funding withdrawals, banks can either sell government bonds or productive capital. Government bonds can be sold at the original value.[12] However, capital is sold at $(1 - \alpha^0)q_t$, which reflects both the lower endogenous capital value ($q_t < q_{t-}$) after a crisis, and also the temporary market pressure as determined by the parameter $\alpha^0 > 0$. The latter captures the market illiquidity in a financial crisis, which gets amplified into lower $q_t$ due to

[9] Since households do not know which bank suffers from the exposure, they do not differentiate across banks.

[10] The assumption of $\beta < 1$ is critical for banks' liquidity management on the asset side. If $\beta = 1$, asset-side and liability-side liquidity is exactly matched for any public liquidity holding, so banks are indifferent to any level of public liquidity holding, similar to the cancellation of inside and outside money in Brunnermeier and Sannikov (2016).

[11] See Brunnermeier and Sannikov (2014), Rampini and Viswanathan (2019), etc. Microfoundations include asymmetric information and moral hazard problems.

[12] The above mechanism has broader interpretations in reality and is closely related to the Treasury repo market. In practice, bank Treasury positions are partially funded by repo and partially funded by non-repo financing. In the 2008 crisis, Treasury repo rate remains stable (Copeland, Martin and Walker, 2014). The model mechanism can be broadly interpreted as capturing both of the following trades: (a) banks sell non-repo-financed Treasuries for cash; (b) banks raise more secured financing by posting as collateral the non-repo-financed Treasuries. It is likely that both (a) and (b) are at play in reality and the model concisely captures both of them.

endogenously lower bank equity after the crisis.

The following lemma summarizes the events in a crisis and the existence of sudden funding withdrawals only in crises.

**Lemma 1.** *When a crisis shock occurs at t, households withdraw the maximum possible amount of their non-deposits (i.e., $\beta$ fraction of total non-deposits). Banks have to sell assets to fulfill the funding withdrawals. In normal times, there is no equilibrium of sudden household funding withdrawals.*

In a crisis shock $dN_t$, bank equity endogenously declines and banks reduce their risky capital holding, which lowers productivity and thus capital value $q_t$. The dynamics of capital price $q_t$ is

$$\frac{dq_t}{q_{t-}} = \mu_t^q dt + \sigma_t^q dZ_t - \kappa_{t-}^q dN_t, \tag{5}$$

where $\mu_t^q$, $\sigma_t^q$, and $\kappa_{t-}^q$ are all endogenously determined, and $\kappa_{t-}^q$ reflects the severity of the crisis. Then the return of capital held by bank $j$ is

$$d\bar{R}_{j,t}^K = \underbrace{\frac{d(q_t k_{j,t})}{q_t k_{j,t}}}_{\text{“capital gain”}} + \underbrace{\frac{(\bar{A} - \phi(\mu_t^K))k_{j,t}}{q_t k_{j,t}} dt}_{\text{“dividend”}}. \tag{6}$$

Similarly, the return of capital held by an individual household is denoted as $d\underline{R}_{j,t}^K$. The expression for $d\underline{R}_{j,t}^K$ is the same as (6) except for a lower productivity $\underline{A}$. Since bankers dominate households in their return of capital, to generate a non-singular distribution of relative wealth, I assume that bankers become households at the rate of $\eta > 0$.[13]

The government issues $B_t K_t$ amounts of bonds in total ($K_t$ is the total amount of capital), and back these bonds by taxation and holding non-deposit bank debt that are worth

$$A_t K_t = (B_t - \bar{B}) K_t \tag{7}$$

Since we have a consolidated government in mind, these government bonds are public liquidity a la Holmström and Tirole (1998). This is a convenient modeling approach that captures quantitative easing policies expanding public liquidity $B_t$ backed by high-quality private assets.

The return of government bonds is denoted as

$$dR_t^B = r_t^B dt, \tag{8}$$

[13]This transition is accounted for by the banker's optimization problem. The economy will not "run out of bankers" since there is a continuum of them. In the model, what matters is banker wealth, which has a stationary distribution.

and the return of insured deposits is denoted as as

$$dR_t^D = r_t^D dt, \tag{9}$$

The aggregate return of non-deposits is denoted by

$$dR_t^{ND} = r_t^{ND} dt - \kappa_{t-}^{ND} dN_t, \tag{10}$$

where the average loss $\kappa_{t-}^{ND} = \theta(1-\beta)$ as indicated by the bank liquidation process.

Deposits provide transaction convenience to households: for household deposits holding $y^D$ (as a fraction of wealth), they obtain additional wealth growth by $v_D(y^D)$, which is a concave function, so the marginal benefit decreases. We can think of $v_D(y^D)$ as reflecting the transaction value from holding deposits. The reason for not using the more standard money-in-the-utility approach is to keep the consumption rule of households and bankers the same, which isolates financial frictions as the driver of discount rate and capital value. On the other hand, to provide the deposit convenience to households, banks incur a convex cost $c_D(x^D)$, which reflects the costs of setting up deposits branches, recruiting employees, and advertising to attract customers, etc. The convex functional requirement guarantees an interior solution and follows Hugonnier and Morellec (2017) and Bolton et al. (2021).

The government spends $g_t K_t dt$ amount at time $t$. The government balances its budget at each $t$, leading to the equation

$$\underbrace{E_{t-}[d(B_t K_t)]}_{\text{govt bond issuance}} + \underbrace{d\mathcal{T}_t}_{\text{tax income}} + \underbrace{E_{t-}[A_t K_t dR_t^{ND}]}_{\text{interest income}} = \underbrace{E_{t-}[d(A_t K_t)]}_{\text{change in portfolio}} + \underbrace{r_t^B B_t K_t dt}_{\text{interest payment}} + \underbrace{g_t K_t dt}_{\text{govt spending}} \tag{11}$$

where $d\mathcal{T}_t$ is the lump-sum taxation that charges a fix fraction $\tau_{t-}dt$ proportionate to the wealth of agents. I use a weaker version of budget balance and only require the balance in expectation, so taxation is always a deterministic term. Thus, taxation does not affect economic allocations or uncertainty.

The government debt growth follows the process below

$$dB_t = -\theta_B(B_t - \bar{B})dt + \kappa^B dN_t \tag{12}$$

where $\kappa^B > 0$ is the contingent expansion of government debt during a crisis. Starting from $B_0 = \bar{B}$, the level of the government debt spikes during crises but goes down in normal times towards the stationary steady state $\bar{B}$. A small $\theta_B$ will generate a highly persistent government debt process as in the data.

## 3.2. Household and Banker Decisions

With the model setup, I derive the individual optimization problems for households and bankers.

### Households

Denote the wealth of an individual household as $w^h_{j,t}$. Household $j$ chooses the consumption rate $c^h_{j,t}$, capital holding $y^K_{j,t}$, government bond holding $y^B_{j,t}$, deposits holding $y^D_{j,t}$, and non-deposits holding $y^{ND}_{j,t}$, all as fractions of wealth. Denote the tax rate as $\tau_{t-}$. In a crisis, banks sell capital at below the post-crisis equilibrium price which are bought by households. Banks cannot purchase the fire-sold capital, because in order to do so, they need to raise non-deposits from households. However, households cannot differentiate banks and it is their dominant strategy not to lend to any bank. In the crisis, households purchase the fire-sold capital and gain fire-sale benefits. Thus, the discount on capital in a crisis is merely a transfer of wealth from bankers to households. I assume that households divide this benefit proportionate to their wealth, denoted by $\kappa^{fs}_{t-}$.

Due to risk aversion, households optimally choose to diversify across a continuum of banks. This diversification implies that the non-deposit return for each household is the same $dR^{ND}_t$. Thus, the budget constraint of a household is

$$\begin{aligned}\frac{dw^h_{j,t}}{w^h_{j,t-}} = & \underbrace{y^K_{j,t-} d\underline{R}^K_{j,t}}_{\text{capital return}} + \underbrace{y^B_{j,t-} dR^B_t}_{\text{govt debt return}} + \underbrace{y^D_{j,t-} dR^D_t}_{\text{deposit return}} + \underbrace{v^D(y^D_{j,t-})dt}_{\text{deposit convenience}} \\ & + \underbrace{y^{ND}_{j,t-} dR^{ND}_t}_{\text{non-deposit return}} - \underbrace{\tau_{t-} dt}_{\text{tax}} + \underbrace{\kappa^{fs}_{t-} dN_t}_{\text{firesale benefits}} - \underbrace{\frac{c^h_{j,t-}}{w^h_{j,t-}} dt}_{\text{consumption}}\end{aligned} \tag{13}$$

subject to non-negative constraints $y^K_{j,t-} \geq 0$, $y^B_{j,t-} \geq 0$, $y^D_{j,t-} \geq 0$, $c^h_{j,t-} \geq 0$, and balance sheet identity $y^K_{j,t-} + y^B_{j,t-} + y^D_{j,t-} + y^{ND}_{j,t-} = 1$. The notation $t-$ means that the decision is made conditional on the information up to $t-$, right before a crisis.

### Bankers

Let $w^b_{j,t}$ be the wealth of an individual banker $j$. Each banker can invest in productive capital, government bonds and borrow from households with either insured deposits or non-deposits. Denote banker $j$'s portfolio choice in productive capital as $x^K_{j,t}$ and in government bonds as $x^B_{j,t}$. Denote bank deposits as $x^D_{j,t}$ and non-deposits as $x^{ND}_{j,t}$. All of the above portfolio decisions are expressed as fractions of banker wealth.

In a crisis $dN_t = 1$, according to Lemma 1, banks have to liquidate assets to fulfill the funding withdrawal of amount $\beta x^{ND}_{j,t-}$. The optimal strategy is to first liquidate all the government bonds, and if still not enough, liquidate capital. I denote the required liquidity via capital sales as

$$\Delta x_{j,t-} = \max( 0 \, , \underbrace{\beta x^{ND}_{j,t-}}_{\text{funding withadrawls}} - \underbrace{x^{B}_{j,t-}}_{\text{liquidity holding}} ) \tag{14}$$

To meet this liquidity need, banks have to sell $\Delta x_{j,t-}/((1-\alpha^0)q_t)$ units of capital and lose $\alpha^0 \Delta x_{j,t-}/(1-\alpha^0)$ due to the illiquidity discount, beyond the losses due to capital value drop. With partial liquidation $\beta < 1$, one unit of government bond can insure more than one unit of non-deposit funding. Therefore, banks value public liquidity for its protection against liquidity shocks.

Thus, bank budget constraint is

$$\begin{aligned} \frac{dw^b_{j,t}}{w^b_{j,t-}} = & \underbrace{x^K_{j,t-} d\bar{R}^K_{j,t}}_{\text{capital return}} - \underbrace{\frac{\alpha^0}{1-\alpha^0}\Delta x_{j,t-}}_{\text{illiquidity discount}} + \underbrace{x^B_{j,t-} dR^B_t}_{\text{govt debt return}} - \underbrace{x^D_{j,t-} dR^D_t}_{\text{deposit return}} - \underbrace{c_D(x^D_{j,t-})dt}_{\text{deposit cost}} \\ & - \underbrace{x^{ND}_{j,t-} dR^{ND}_t}_{\text{non-deposit return}} - \underbrace{\tau_{t-} dt}_{\text{tax}} - \underbrace{\frac{c^b_{j,t-}}{w^b_{j,t-}} dt}_{\text{consumption}} \end{aligned} \tag{15}$$

subject to non-negative constraints $x^K_{j,t-} \geq 0$, $x^B_{j,t-} \geq 0$, $x^D_{j,t-} \geq 0$, $x^{ND}_{j,t-} \geq 0$, and the balance sheet identity $x^K_{j,t-} + x^B_{j,t-} = 1 + x^{ND}_{j,t-} + x^D_{j,t-}$.

### *3.3. Equilibrium Definition*

I restrict the equilibrium definition to a Markov equilibrium. The model has three state variables. The first is the wealth share of bankers

$$w_t = \frac{W^b_t}{W^b_t + W^h_t}, \tag{16}$$

where $W^b_t$ is the total wealth of bankers and $W^h_t$ is the total wealth of households. The second one is the relative amount of government bonds $B_t$ (note that the total value is $B_t K_t$). The third one is the aggregate productive capital $K_t$. Since all aggregate quantities are scaled by $K_t$, I can solve for ratios of aggregate quantities over $K_t$ as a function of $w_t$ and $B_t$ only.

Denote the share of capital owned by bankers as

$$\psi_t = \frac{x_t^K W_t^b}{x_t^K W_t^b + y_t^K W_t^h}. \tag{17}$$

Then the aggregate output is

$$Y_t = (\psi_t \bar{A} + (1-\psi_t)\underline{A})K_t. \tag{18}$$

Because $\bar{A} > \underline{A}$, the productivity of the economy is higher with larger $\psi_t$.

Homogeneous preferences imply that individual consumption-wealth ratio and portfolio choices are independent of individual wealth level. As a result, $\hat{c}_t^b := c_t^b/w_t^b$, $x_t^K$, $x_t^B$, $x_t^D$ are the same for all bankers, and $\hat{c}_t^h := c_t^h/w_t^h$, $y_t^K$, $y_t^B$, $y_t^D$ are the same for all households. Then the dynamics of aggregate variables are

$$\frac{dW_t^b}{W_t^b} = \frac{dw_t^b}{w_t^b} - \eta dt \tag{19}$$

$$\frac{dW_t^h}{W_t^h} = \frac{dw_t^h}{w_t^h} + \eta \frac{W_t^b}{W_t^h} dt, \tag{20}$$

where the second terms in equations (19) and (20) are due to the transition of bankers to households.

The following formalizes the equilibrium definition.

**Definition 1** (Equilibrium)**.** *An equilibrium is a set of prices and allocations that depend only on the state variables $w_t$, $B_t$ and $K_t$. These include the price of capital $q_t$, price jump during crisis $\kappa_{t-}^q$, investment decision $\mu_t^K$, bank and household portfolio choices, and consumption/wealth ratios $\hat{c}_t^h$ and $\hat{c}_t^b$, such that*

- *Consumption, investment, and portfolio choices are optimal.*
- *The capital goods market clears*

$$W_t^b x_t^K + W_t^h y_t^K = q_t K_t \tag{21}$$

- *The public liquidity market clears*

$$W_t^b x_t^B + W_t^h y_t^B = B_t K_t \tag{22}$$

- *The aggregate wealth accounting identity is satisfied*

$$W_t^h + W_t^b = q_t K_t + \bar{B} K_t \tag{23}$$

*where the second term $\bar{B}K_t$ appears because the wealth is defined as pre-tax wealth that has not deducted future tax liabilities, which are worth $B_tK_t - A_tK_t = \bar{B}K_t$.*

- *Government budget constraint*

$$\bar{B}E_{t-}[\frac{dK_t}{dt}] + \tau_{t-}(W_t^b + W_t^h) + A_tK_t(r_t^{ND} - \lambda\kappa_{t-}^{ND}) = r_t^B B_tK_t + g_tK_t \tag{24}$$

*which is a simplified version of (11) after plugging in (7) and notations of aggregate wealth.*

- *Market clearing condition for the insured deposits,*

$$x_t^D W_t^b = y_t^D W_t^h \tag{25}$$

- *Resource constraint*

$$\left(\psi_t\bar{A} + (1-\psi_t)\underline{A}\right)K_t = \hat{c}_t^b W_t^b + \hat{c}_t^h W_t^h + \phi(\mu_t^K)K_t + g_tK_t, \tag{26}$$

*where total output equals to the sum of consumption, investment, and government spending.*

- *Jumps in a crisis solve the fixed-point problem*

$$q(w_t, B_t) - q(w_{t-}, B_{t-}) = -\kappa_{t-}^q q(w_{t-}, B_{t-}) \tag{27}$$

*where the post-crisis state $w_t$ endogenously depends on both $w_{t-}$ and $\kappa_{t-}^q$.*

Market clearing is not needed for non-deposits because of Walras's Law. Furthermore, to highlight the role of bank demand for public liquidity, I assume that $\delta_B \in (0,1)$ fraction of government bonds are held by the banking sector, while the household sector passively subsumes the rest. Without this assumption, the economy may feature a bang-bang solution of government bond allocation and needs further ingredients to generate a reasonable allocation of public liquidity.

# 4. Model Mechanism and Calibration

In this section, I provide model solutions and illustrate the key mechanisms that connect public liquidity to both asset prices and the macroeconomy. The key result is that public liquidity crowds out private liquidity (deposits), increases bank leverage, but reduces financial fragility. Furthermore, a larger supply of public liquidity increases bank lending and boosts the price of capital. The key asset price that reflects the marginal value of public liquidity is the liquidity premium.

To simplify notations, I will mostly omit subscripts $t$ and $t-$. The detailed step-by-step derivations are provided in the appendix.

## *4.1. Consumption and Portfolio Choices*

With log utility, household problem can be equivalently written as

$$
\begin{aligned}
\max_{y^K,y^B,y^D,y^{ND},c^h} \; & \underbrace{y^K(\mu^R + \frac{\underline{A}}{q} - r^{ND})}_{\text{excess capital return}} + \underbrace{y^B(r^B - r^{ND})}_{\text{excess govt bond return}} + \underbrace{y^D(r^D - r^{ND})}_{\text{excess deposit return}} \\
& + \underbrace{r^{ND}}_{\text{non-deposit return}} + \underbrace{v_D(y^D)}_{\text{deposit utility}} + \underbrace{\rho \log(c^h)}_{\text{consumption utility}} - \underbrace{\frac{c^h}{w^h}}_{\text{consumption}} - \underbrace{\frac{1}{2}\left(y^K(\sigma^q + \sigma^K)\right)^2}_{\text{risk aversion to vol}} \\
& + \underbrace{\lambda\theta \log(1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs})}_{\text{expected utility in case of bankruptcy}} + \underbrace{\lambda(1-\theta)\log(1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs})}_{\text{expected utility in case of surviving banks}}
\end{aligned}
\tag{28}
$$

subject to the balance sheet identity $y^K + y^B + y^D + y^{ND} = 1$. The non-dividend component of capital return is defined as

$$
\mu^R \equiv \mu^q - \delta + \mu^K + \sigma^K\sigma^q - \frac{\phi(\mu^K)}{q} \tag{29}
$$

The last line in (28) expresses the two cases of crisis shock exposure: with a small probability $\theta$, an individual household's capital is destroyed, so the percentage decline in wealth is $y^K$; with probability $1-\theta$, the individual household's capital is not destroyed but subject to the endogenous price decline, causing a drop in wealth by $y^K\kappa^q$. In a crisis, the household is also subject to losses from non-deposits, $y^{ND}\kappa^{ND}$, and gains from bank capital sales, $\kappa^{fs}$.

The problem of the bank is as follows:

$$
\begin{aligned}
\max_{x^K,x^B,x^D,x^{ND},c^b} \; & \underbrace{x^K(\mu^R + \frac{\bar{A}}{q} - r^{ND})}_{\text{excess capital return}} + \underbrace{x^B(r^B - r^{ND})}_{\text{excess govt bond return}} - \underbrace{x^D(r^D - r^{ND})}_{\text{deposit interest payment cost}} \\
& + \underbrace{r^{ND}}_{\text{non-deposit return}} - \underbrace{c_D(x^D)}_{\text{deposit production cost}} + \underbrace{\rho \log(c^b)}_{\text{consumption utility}} - \underbrace{\frac{c^b}{w^h}}_{\text{consumption}} - \underbrace{\frac{1}{2}\left(x^K(\sigma^q + \sigma^K)\right)^2}_{\text{risk aversion to vol}} \\
& + \lambda\left(\underbrace{\theta \log(\varepsilon)}_{\text{expected utility with bankruptcy}} + \underbrace{(1-\theta)\log(1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}(\beta x^{ND} - x^B))^+}_{\text{expected utility without bankruptcy}}\right)
\end{aligned}
\tag{30}
$$

subject to the balance sheet identity $x^K + x^B = 1 + x^D + x^{ND}$.

The notable difference from household optimization is that bank deposits production is subject to the convex operation cost $c_D(x^D)$. Furthermore, as illustrated in the last line of (30), with probability $\theta$, the bank is subject to full exposure to the crisis shock and obtains a residual amount of equity, $\varepsilon > 0$. With probability $1-\theta$, the bank is not directly exposed to the crisis shock, but suffers from the endogenous capital value decline, $x^K \kappa^q$, and as well as the market illiquidity losses from $\alpha^0$ and fire-sold capital. It is in the latter that holding government bonds matters: by increasing the government bond holding, the bank can reduce the net funding withdrawal on capital, $\beta x^{ND} - x^B$, which dampens the drop in bank equity.

In both household and banker optimization problem of (28) and (30), the optimal consumption rule is to consume $\rho$ fraction of wealth, i.e.,

$$c^h = \rho w^h, \quad c^b = \rho w^b \tag{31}$$

## *4.2. Public Liquidity on Financial Fragility and Private Liquidity*

To derive theoretical insights, I assume that asset price dynamics, $dq_t/q_t$, is given. Then we obtain the following results:

**Proposition 1** (Public Liquidity, Financial Fragility, and Private Liquidity)**.** *Given asset price dynamics, for any set of state variables $(w, K)$, a larger public liquidity supply $B$ reduces financial fragility $\Delta x$, but it crowds out private liquidity (bank deposits $x^D$) and increases bank leverage.*

The results in Proposition 1 can be best understood via the liquidity insurance view. When there is a larger public liquidity supply $B$, the cost of holding public liquidity decreases, and the equilibrium bank holding of public liquidity expands, which implies that banks are better liquidity insured. Therefore, banks expand their balance sheet more and take higher leverage. On the liability side composition, with better liquidity insurance via holding government bonds, banks value the stickiness of insured deposits less and thus reduce the supply of insured deposits.

The crowding-out effect of public liquidity on bank deposits matters in the model because households are less insured against bankruptcy risks. If the bankruptcy risk $\theta$ becomes larger, this reduction in insurance will become more prominent. Furthermore, in reality, deposits are a stable source of funding that has synergy with bank lending (not modeled). This is another reason why we should care about the crowding-out effect.

In Proposition 1, due to the difficulty of endogenous jumps in the equilibrium, results are conditional on asset price dynamics. Nevertheless, all of the results go through in the numerical solutions that incorporate the whole equilibrium feedback, including the fixed-point problem of (27).

## 4.3. Public Liquidity Expands Bank Lending and Boosts Capital Value

Despite less insured bank deposits, I next show that larger public liquidity actually expands bank lending and boosts capital value.

**Proposition 2** (Public Liquidity and the Real Economy)**.** *Given asset price dynamics, for any set of state variables $(w, K)$, a larger public liquidity supply $B$ increases bank lending $x^K$, boosts capital value $q$, and increases the total productivity.*

Proposition 2 implies that the supply of public liquidity affects the real economy. When more public liquidity is available, banks are better liquidity insured and charge a low risk premium over risky productive capital, which boosts the capital price. Since banks increase their capital holding and they are more efficient owners of capital in the economy, the result is a higher total productivity in the economy.

In Proposition 2, the conclusions are partial equilibrium and conditional on the asset pricing dynamics. In the full numerical solution, after accounting for the equilibrium feedback of capital return, volatility, and jumps, the conclusion of Proposition 2 still holds.

## 4.4. Public Liquidity and the Liquidity Premium

To construct a measure of pure liquidity value, I define $r^f$ as the illiquid risk-free rate implied by the bank's SDF, and assume that only a portion $(1-\pi) \in (0,1)$ of this illiquid risk-free asset can be sold during a crisis. Then I define the liquidity premium as

$$\ell = r^f - r^B \tag{32}$$

Then we have the following results:

**Proposition 3** (Liquidity Premium)**.** *The liquidity premium $\ell$ is priced by banks,*

$$\ell = \underbrace{\lambda}_{\text{crisis frequency}} \cdot \underbrace{(1-\theta)}_{\text{prob of survival}} \cdot \underbrace{\frac{\frac{\alpha^0}{1-\alpha^0}1_{\Delta x>0}}{1 - x^K \kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}}_{\text{marginal value of liquidty insurance}} \cdot \underbrace{\pi}_{\text{liquidity differential}}, \tag{33}$$

*Given asset price dynamics (the same assumption as Proposition 1 and 2), a larger public liquidity supply $B$ reduces the liquidity premium.*

According to Proposition 3, as $\pi \to 0$, the illiquid asset becomes as liquid as government bonds, and the liquidity premium shrinks to zero, while the dynamic variations of $\ell$ remain the same. Thus, we shall treat $\pi$ as the scaling parameter that allows us to match the data. In reality, the liquidity premium spread is always constructed with reference to a less liquid assets, but the level of liquidity differential depends on the specific asset.

Equation (33) suggests that public liquidity is only valuable if banks are exposed to an actual liquidity problem with illiquid capital ($\Delta x > 0$). In this case, the liquidity value is positively related to crisis frequency $\lambda$, the likelihood of non-bankruptcy distress $1 - \theta$, the illiquidity of capital $\alpha^0$, and the liquidity differential between the illiquid risk-free asset and government bonds, $\pi$.

Intuitively, as public liquidity supply $B$ increases, the vulnerability of banks, as measured by $x^K \kappa^q + \frac{\alpha^0}{1-\alpha^0} \Delta x$, decreases. Consequently, the risk adjustment for liquidity shocks is smaller, leading to a smaller liquidity premium.

Apart from the liquidity supply $B$, the endogenous state variable $w$ also has a significant impact on the liquidity premium, interpreted as the liquidity demand effect. In general, when banker wealth share $w$ is small, bank leverage is high ($x^K$ is larger), and banks are more fragile. As a result, according to (33), the marginal value of liquidity insurance increases, leading to a larger liquidity premium.

Finally, we can view equation (33) from an individual bank's perspective. For an individual bank, equation (33) states the impact of the market-determined liquidity premium on the bank's portfolio choices. The direct impact is on government bonds holding $x^B$. Keeping all else equal, according to equation (33), $x^B$ is larger when the liquidity premium decreases. The idea is that a lower liquidity premium means it is less costly to hold government bonds, which increases the individual bank's liquidity insurance. The increase of $x^B$ reduces $\Delta x$ and makes the funding withdrawal less disruptive, which encourages more bank risk taking through expanding the balance sheet and holding more productive capital.

## *4.5. Why Public Liquidity Matters?*

Next, I illustrate that the supply of government bonds matters in the model because of three financial frictions, and all of them are essential.

**Proposition 4** (The Role of Financial Frictions)**.** *Ricardian equivalence (government bond supply $B$ does not affect the real economy) holds when any one of the following conditions are satisfied:*

- *No equity friction: Banks can freely issue equity.*
- *No runnable deposits: all deposits are sticky with $\beta = 0$.*

- *No illiquidity discount:* $\alpha_0 = 0$.

*Proof.* Refer to Appendix A.9. □

Proposition 4 shows that the model has the minimum elements necessary to connect the public liquidity supply to the real economy. Absent any one of them, Ricardian equivalence holds, where government bond supply has no impact on equilibrium allocations and asset prices. Therefore, the effects of public liquidity in this model arise from the liquidity insurance mechanism introduced by combining the above three frictions.

The role of each friction is different. The equity friction is the fundamental friction that makes financial sector equity affect the macroeconomic dynamics. The runnable deposit friction triggers liquidity problems in the financial sector. Finally, the illiquidity discount materializes the impact of crisis shocks and avoids the equilibrium where capital is immediately sold back from households to bankers without any allocative inefficiency.

## *4.6. Nonlinearity of Crisis Severity*

Financial frictions generate nonlinearity, which implies that the severity of a crisis varies dramatically across states. Figure 2 illustrates such state dependence, with the price of capital $q$ in the vertical axis (directly related to the productivity of the economy) and the state variable $w$ in the horizontal axis. The horizontal axis is divided into two regions by the dotted line. In the left region, the output sensitivity to bank equity is greater than zero, also reflected in the non-zero slope of the price of the capital curve (solid line). In the right region, the output sensitivity to bank equity is zero. As a result, bank equity has an asymmetric impact on the real economy, consistent with the empirical results in Adrian, Boyarchenko and Giannone (2019).

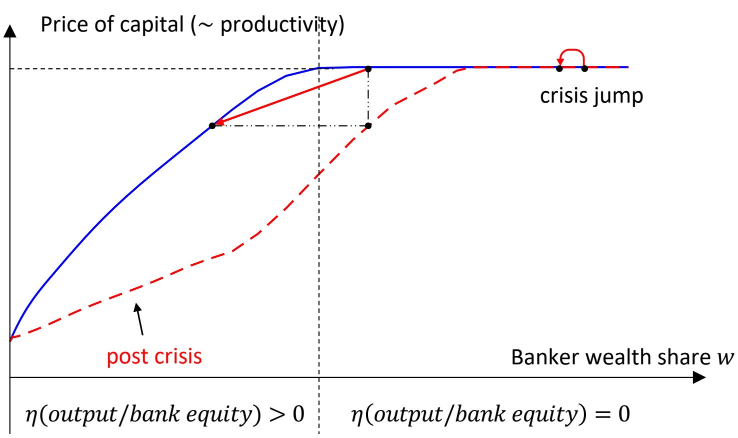



Figure 2. **State Dependence of Crisis Severity and Output Sensitivity.**

The dashed curve represents the immediate post-crisis price of capital, conditional on a pre-crisis banker wealth share $w$. If the pre-crisis bank equity share is slightly above the dotted boundary, a crisis shock reduces banker wealth share to below the dotted boundary, and therefore, productivity drops. However, if the pre-crisis bank equity share is well above the dotted boundary, the post-crisis bank equity share is also above the boundary, and productivity remains the same.

## *4.7. Calibration*

I map the model to the U.S. economy. Using the flow of funds terminology, banks in the model represent bank holding companies (parent only), private depository institutions, and broker-dealers.

Households in the model are interpreted as a combination of individuals and pass-through institutions, including money market funds (MMFs), mutual funds, and pensions. This interpretation creates a consistent mapping between bank liquidity problems in the model, and those in reality. During the 2008 financial crisis, MMFs and other financial institutions dramatically cut off funding to the repo market, so the net borrowers from the repo market suffered, including broker-dealers and off-balance-sheet vehicles supported by commercial banks.

In the model, banks have both insured deposits and non-insured liabilities. Mapping to the data, the insured deposits, including checking and savings deposits under the FDIC insurance limit. Among the non-insured liabilities, runnable deposits in the model (the $\beta$ fraction of non-deposits) map to short-term uninsured liabilities, mainly in the form of repos and asset-backed commercial papers (ABCP). Indeed, during the 2008 financial crisis, repo and ABCP markets declined by several trillion dollars. In the data, the fraction of runnable liabilities over total bank liabilities is 0.25 according to the flow of funds data from 2001 to 2018. Furthermore, according to Egan, Hortaçsu and Matvos (2017), "on average, uninsured deposits account for just over half (53.36 percent) of total deposits, while total deposits account for 77 percent of liabilities ...". Thus, the fraction of insured deposits over total liability is $0.77*0.53 \approx 40\%$, which will be used as a target for deposits production cost. This also implies that the fraction of runnable liabilities over non-insured liabilities is $0.25/(1-0.4) = 42\%$. Thus, I set $\beta = 42\%$ as the average of banks' runnable liabilities over total non-insured debt.

The short-term price pressure $\alpha^0$ directly corresponds to the haircut increase during the financial crisis, because it reflects how much the assets can be liquidated on the market, and the change is quite transitory. According to Bai, Krishnamurthy and Weymuller (2018), "the loan haircut in the secondary market is relatively constant and remains less than 5% in normal times, but then increases to as high as 40% during the 2008 to 2009 crisis." Given that loans are about 60% of the non-cash and non-Treasury assets on the balance sheets of bank holding companies, I estimate an aggregate market price pressure on assets excluding public liquidity as 0.6*(0.4-0.05) = 21%. Changes in haircuts of other assets are neglected because they are much smaller compared to the loan haircut spikes.

The crisis frequency is set as $\lambda = 4\%$ to match the average frequency of financial crises in the data (Jordà, Schularick and Taylor, 2013). I follow the macroeconomics literature to set annual depreciation rate $\delta = 0.1$ (Gertler and Kiyotaki, 2010), annual time discount rate $\rho = 4\%$ (Gertler and Kiyotaki, 2010), and investment adjustment cost $\chi = 3$ (He and Krishnamurthy, 2014), with an investment adjustment cost function as

$$\phi(\mu^K) = \mu^K + \frac{\chi}{2}(\mu^K - \delta)^2. \tag{34}$$

According to the flow of funds, among Treasuries supplied to the private sector, the fraction held by broadly-defined banks (including U.S.-chartered depository institutions, foreign banking offices in the U.S., credit unions, Security brokers and dealers, and holding companies) is 20% from 1970 to 2016. Therefore, I set $\delta_B = 0.2$ in the model.

Bankruptcy leftover $\varepsilon$ is set to a small value $10^{-3}$, and the aggregate capital destruction in a crisis shock is set to $\theta = 10^{-5}$. Thus, the effect of the financial crisis shock is not directly coming from capital destruction, and banks are allowed to have very low capitalization before taken over by the government for liquidation.

I specify the deposits production cost function as $c_D(x) = \frac{1}{2}\bar{c}_D \cdot x^2$, and the household convenience over holding deposits as $v_D(y^D) = \beta_D \log(y^D/\underline{y}^D)$. For simplicity, the government spending is taken as a constant fraction over capital, $g_t = \bar{g}$. Notice that since productivity goes down in a crisis, this government spending rule implies a higher spending/GDP ratio in a crisis but lower spending/GDP ratio in normal times, consistent with the counter-cyclical patterns in the data.[14]

Next, I jointly estimate the parameters $(\bar{A}, \underline{A}, \sigma^K, \eta, \beta_D, \underline{y}^D, \theta_B, \kappa_B, \bar{B}, \bar{g}, \pi)$ to match the following moments:

- The model-implied public liquidity/GDP process, $B_t/(\psi_t\bar{A} + (1-\psi_t)\underline{A})$, is matched to the data counterpart, where public liquidity is the total of bank reserves and government debt minus intra-government holding and Fed holdings of Treasuries (more details are provided in Section 5.2). From 1970 to 2020, the minimum of this ratio is 0.18, while the average

[14]My model can also incorporate richer fiscal dynamics although that is not the focus of the paper. For richer government expenditure processes, refer to Fernández-Villaverde (2010) and Croce et al. (2012).

is 0.38. The average change of this ratio at the onset of the 2008 crisis and the 2020 crisis is about 0.1. These three moments are mainly to discipline $(\theta_B, \kappa_B, \bar{B})$.

- The average decline of output in a crisis is matched to 8%. According to Jordà, Schularick and Taylor (2013), the 5-year decline in GDP from the date of crisis of around 8%. Note that in the data, crises unfold over time, while in the model, crisis happened immediately.
- The average of productivity $\psi_t \bar{A} + (1 - \psi_t)\underline{A}$ is matched to 14% following He and Krishnamurthy (2014).
- The average liquidity premium $\ell_t$ is 0.3%, where the details of the liquidity premium construction in the data are provided in Section 2.
- The average deposit spread, $r^{ND} - r^D$, is 1.5%, according to the call report data. The deposit spread is defined as the volume-weighted FFR-checking and FFR-savings spread.
- The fraction of insured deposits over total liability is 40%. Discussions about this value are in earlier paragraphs of this section.
- The volatility of the real GDP growth is targeted to 2.1%, which is the real-GDP growth volatility for the U.S. from 1970 to 2016.
- The household convenience over holding deposits, $v_D(y^D)$ is on average zero. This is to achieve a similar result to the liquidity-in-the-utility setup where the extra liquidity convenience does not affect household wealth growth.
- The average bank equity ratio of the financial sector is set to 10% as in Begenau and Landvoigt (2021).

In the above estimations, there are 11 parameters and 11 moments, so the model is exactly identified. Appendix Table A1 summarizes the calibrated parameter values, and Appendix Table A2 summarizes the estimated parameters and the model and data moments. The time unit is one year for all parameters.

Volatilities of asset prices are non-targeted moments. Liquidity premium volatility is 18 bps in model simulations and 35 bps in the data. Bank leverage volatility is 3.7 in model simulation and 6.4 in the data. These model-implied volatilities are generally smaller than data counterparts, potentially due to other shocks not captured by the model, such as a general change of risk aversion not related to intermediary constraint.

# 5. Pricing of Liquidity

The model provides a general equilibrium explanation of the liquidity premium. In this section, I will use the model to generate a time series of the liquidity premium, and then check whether the resulting time series can explain the variations in the data.

### *5.1. General Equilibrium Pricing of Liquidity*

The idea of the exercise is as follows: the model generates bank equity ratio $1/(x^K(w,B)+x^B(w,B))$ and public-liquidity-to-GDP ratio $B/(\psi(w,B)\bar{A}+(1-\psi(w,B))\underline{A})$, both as functions of the state vector $(w,B)$. In the data, we directly observe bank equity ratio and the public-liquidity-to-GDP ratio, so we can infer the state vector $(w,B)$. Once we know the state vector, the model predicts the liquidity premium, $\ell(w_t,B_t)$, which we can compare with the data counterpart.

The reason for choosing banking sector equity ratio is because it is the core of the liquidity insurance mechanism: banks have a precautionary demand for public liquidity for self-insurance, and such demand will be higher if banks' equity ratio is lower.

The above exercise can be viewed as a test of the liquidity insurance channel. The null hypothesis is that, in reality, banks either do not care about liquidity disruptions, or they are not cautious enough to reserve a buffer of public liquidity. In such a world, we do not expect the model to explain a significant amount of variations in the data. Therefore, a rejection of the null is supportive of the alternative hypothesis that the liquidity insurance mechanism is present.

### *5.2. Data*

For the asset pricing exercise, I use the following data: (i) public liquidity supply/GDP, (ii) financial sector equity ratio, and (iii) the Treasury liquidity premium.

First, I define public liquidity supply as the sum of both bank reserves and Treasuries available to the private sector. The second component is calculated as total Treasuries minus Federal Reserve and intra-government holdings. I use the flow of funds data to calculate the above liquidity supply measure and then divide it by the nominal GDP to get the public liquidity/GDP ratio. This definition directly corresponds to the model interpretation of public liquidity as a liability of a broadly defined government that is available to the domestic economy. The above quarterly data are interpolated to obtain a monthly series.

Second, the financial sector equity ratio data, from He, Kelly and Manela (2017), is an aggregate market equity ratio measure of primary dealers at the bank holding company level. Since most primary dealers were private before the 1970s, the data start from 1970. I scale the equity ratio data uniformly so that the volatility of equity in the data and the model are of the same magnitude. Alternatively, I can also use the equity ratio of all the bank holding companies. Results are similar because primary dealers are representative of all bank holding companies. This equity ratio measure directly corresponds to the $w$ state of the model.

Third, the construction of the liquidity premium has been provided by Section 2.

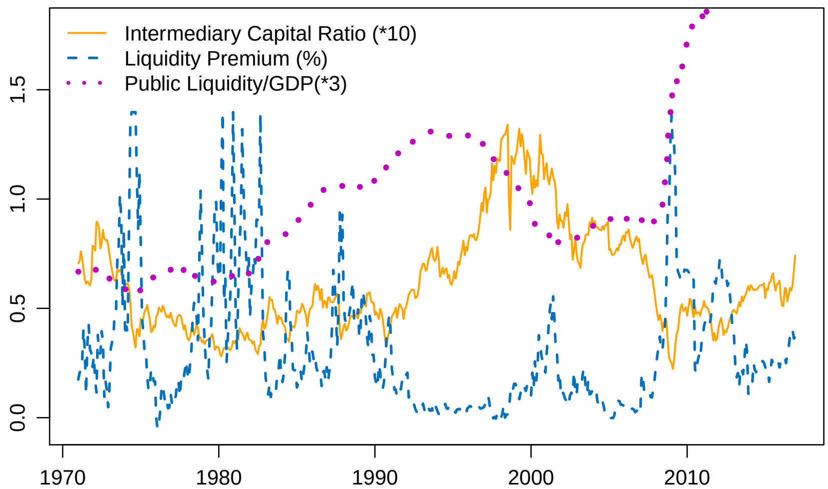



Figure 3. **Time Series Variations of Intermediary Capital Ratio, Public Liquidity Supply/GDP, and the Liquidity Premium**. To illustrate all the data series on the same graph, intermediary capital ratio is multiplied by 10 times, and public liquidity/GDP is multiplied by 3 times. The unit of the liquidity premium is in percentage points.

I illustrate data series three data series in Figure 3. The intermediary capital ratio generally declines during a crisis or recession, while the liquidity premium moves in the opposite direction. Public liquidity/GDP changes at a lower frequency, so it is more difficult to observe the relation with respect to the liquidity premium.

## *5.3. Explanatory Power on the Liquidity Premium*

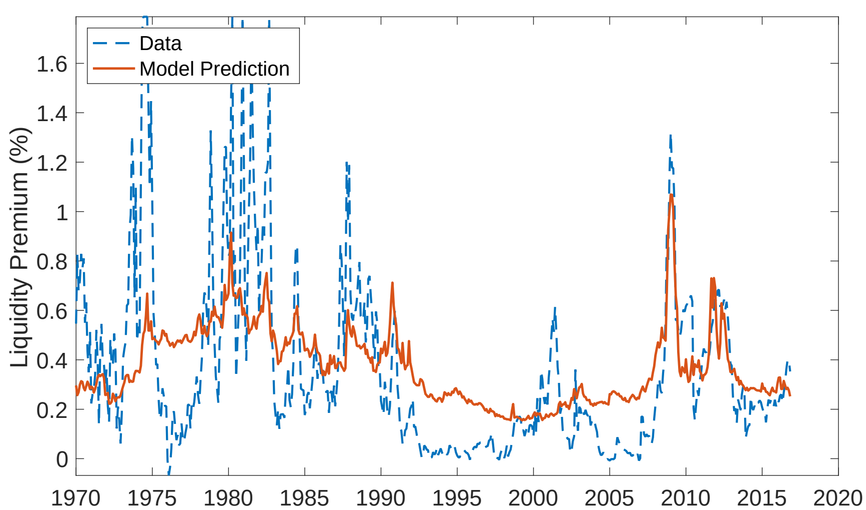



Figure 4. **The Liquidity Premium: Data versus Model.**

The model performs relatively well in generating a reasonable time series of the liquidity premium, as shown in Figure 4. Around the 1980s, there are multiple spikes in the liquidity premium. According to the model, this is because the lower financial sector equity in this period leads to a more vulnerable banking sector and elevated concerns about liquidity disruptions. Then around 2008, the crisis shock drives up the liquidity premium, followed by an increase in public liquidity, which partly contributes to the decline of the liquidity premium in 2009. The main discrepancy between model predictions and the data is seen in the early 2000s, which is probably because of an increased demand for Treasuries by nonbanks that is not captured by the model.

The model predictions (solid line) explain about 40% of the variations in the data (dashed line). However, if the liquidity premium is directly regressed on intermediary capital ratio and the public liquidity supply in linear regression, the $R^2$ is 30%. The critical improvement is the liquidity insurance mechanism, where liquidity demand is nonlinearly related to the capital of banks. A linearized version of equation (33) indicates that $\ell$ is directly related to $x^K$, which is the main driving force of bank leverage in the model. Indeed, in the data, a linear regression of the liquidity premium on intermediary leverage and public liquidity/GDP can generate an $R^2$ of about 40%, similar to the equilibrium model predictions.

To better understand the source of explanatory power on the liquidity premium, I show in Table 2 how the liquidity premium in the data is explained by bank capital ratio, bank leverage and public liquidity/GDP (columns 1-3), and contrast the results with model-implied liquidity premium (columns 4-6).

Comparing column 1 with column 2, we find that the nonlinearity mechanism is important so bank leverage (the inverse of capital ratio) has a much higher explanatory power on the liquidity premium, while in the intermediary asset pricing literature, the typical pricing factor is capital ratio (He, Kelly and Manela, 2017). The critical role of leverage is a key feature of models with endogenous jumps in the state space (see also Krishnamurthy and Li (2020)).

Comparing column 2 with column 3, we find that public liquidity/GDP is consistently negative across settings, confirming the liquidity supply mechanism. The $R^2$ is low on public liquidity/GDP because in the data, public liquidity/GDP is is slow-moving, while the liquidity premium has large fluctuations. This feature has been known in the literature on the convenient yield of government debt (Krishnamurthy and Vissing-Jorgensen, 2012). I show in Appendix C.4 that in a longer sample from 1929 to 2016, the $R^2$ is 12%, and the coefficients on independent variables are very similar. This relatively low explanatory power of public liquidity/GDP on the liquidity premium is not a major concern for the quantitative exercise since it does not serve as the targeted moment. The strength of the liquidity insurance mechanism in the model is disciplined by the fragility of the financial system, as we discussed in Section 4.7.

Table 2: Regressions with Model-Implied Liquidity Premium

| | *Dependent variable: Liquidity Premium* | | | | | |
|---|---|---|---|---|---|---|
| | Data | | | Model-Implied Values | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| capital ratio | $-7.76^{***}$ | | | $-5.60^{***}$ | | |
| | (0.53) | | | (0.14) | | |
| leverage | | $0.03^{***}$ | | | $0.02^{***}$ | |
| | | (0.002) | | | (0.0002) | |
| public liquidity/GDP | $-0.50^{***}$ | $-0.39^{***}$ | $-0.45^{***}$ | $-0.19^{***}$ | $-0.11^{***}$ | $-0.15^{***}$ |
| | (0.08) | (0.07) | (0.10) | (0.02) | (0.01) | (0.04) |
| Observations | 563 | 563 | 563 | 563 | 563 | 563 |
| $R^2$ | 0.30 | 0.42 | 0.04 | 0.76 | 0.96 | 0.02 |

*Notes:* Data are at monthly frequency from 1970 to 2016. Capital ratio is the intermediary capital ratio as in He, Kelly and Manela (2017). Leverage is the inverse of capital ratio. Public liquidity/GDP is the private sector holding of public liquidity, including Treasuries and reserves, as defined in Section 5.2. The dependent variable, the liquidity premium (in percentage points), is constructed following the literature as shown in Section 2.1.

Comparing columns 1–3 with columns 4–6, we find regression coefficients are of the same order of magnitude. The coefficient on public liquidity/GDP is smaller in column 4–6, potentially due to the less elastic demand of Treasuries from other types of investors like pension funds, which are not captured by the model. These non-targeted coefficients validate the model's quantitative relevance.

## *5.4. Impulse Responses*

To better understand the dynamics implied by the model versus the data counterpart, in this subsection, I calculate model-implied impulse response of the liquidity premium to in the average state, and contrast that with the VAR evidence in Section 2.1. In the VAR exercise of Section 2.1, due to orthogonalization, the shock on leverage is orthogonal to capital shocks. To map the model exercise to the data counterpart, I introduce a pure leverage shock (bank capital shock) that reduces $w_t$, but keeps everything else equal. Therefore, the nature of this exercise is very different from that in Section 5.3.

The model-implied impulse response is shown in Figure 5. We find that the liquidity premium in the model has a stronger initial response but the reponse decays faster than the data counterpart. Since the persistence of impulse response in the model is driven by the dynamics of $w_t$, results suggest that the recovery of $w_t$ from a shock might be faster than the data counterpart. Nevertheless, the average responses in the first three years (36 months) are similar.

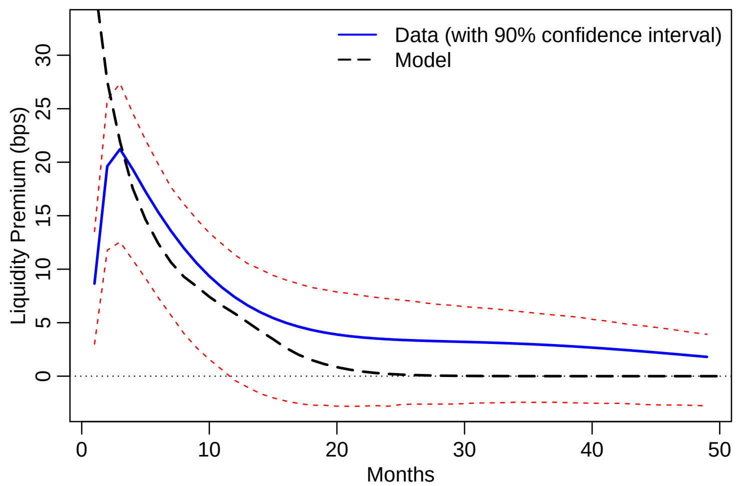



Figure 5. **Impulse Responses of Liquidity Premium to Bank Leverage Shocks: Model v.s. Data.** This figure illustrates the impulse response of the liquidity premium to one standard deviation shock to bank leverage in both the model and data. The data impulse response is the same as Figure 1(a).

# 6. Macro Implications

In this section, I use the model to quantify the impact of public liquidity supply on the real economy during the 2008 financial crisis. The model also speaks to the QE infinity during the recent coronavirus pandemic. At the end of this section, I provide additional quantitative evaluations of the macroeconomic dynamics implied by the model.

## *6.1. Match the 2008 Financial Crisis*

To implement counterfactual analyses, we need to match the GDP dynamics around the 2008 financial crisis. I start the analysis from 2007 and set the state variables $(w_{2007}, B_{2007})$ such that both the liquidity premium and the public liquidity/GDP in the model exactly match the data counterparts. Then I introduce $dZ_t$ shocks in each year and a $dN_t$ shock in

2008, so the GDP percentage changes in the model exactly match the data counterpart.[15] In each year, the liquidity supply $B_t$ is set to match the actual public liquidity/GDP in the data. In other words, if the model-generated change $B_{t+1} - B_t$ is different from the actual changes in the data, there will be an unexpected change of public liquidity supply in the model to exactly match the data.

I detrend the real GDP data series using the Congressional Budget Office (CBO) potential GDP estimate.[16], which results in a 6% total drop below the trend from 2007 to 2009.

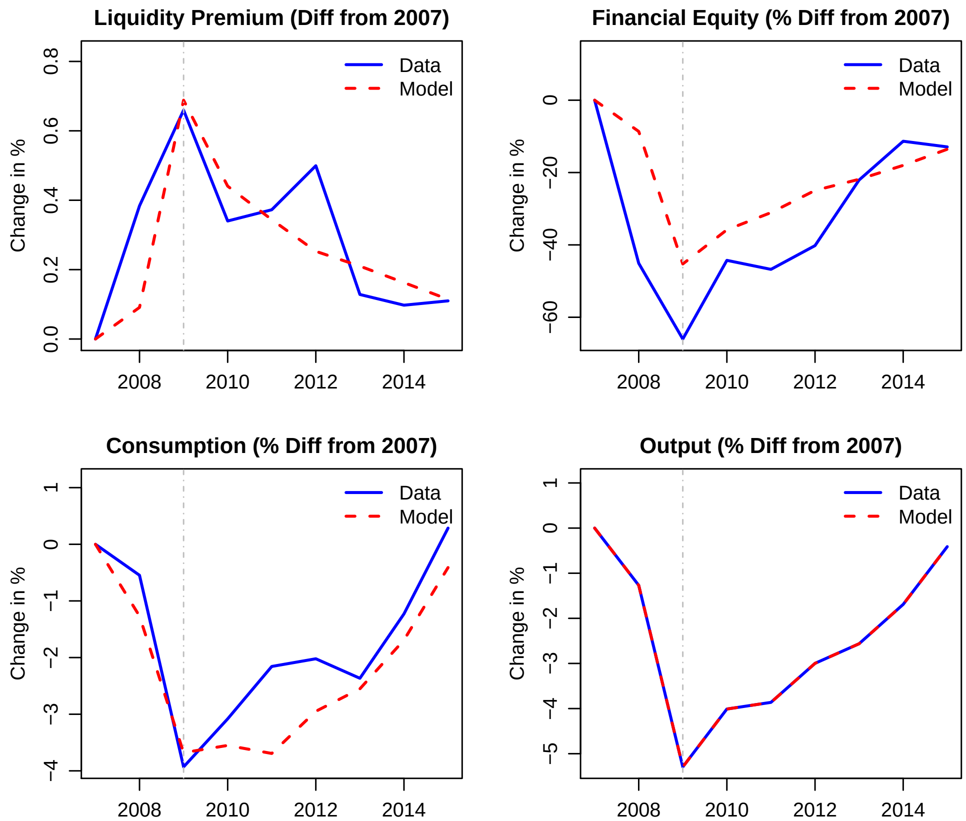



Figure 6. **Data and Model Output around the 2008 Financial Crisis.** In the data, GDP is detrended at the CBO potential GDP (about 1.4%), and the same amount of detrending is performed on total financial sector equity.

Figure 6 compares the model predictions with the data. I separately plot the liquidity premium, financial sector equity, consumption, and output. All quantities are in real terms both in the model and in the data. Financial sector equity is measured as the total market

[15]Note that the size of $dN_t$ is always 0 or 1, but the size of $dZ_t$ can be any number. As a result, the $dZ_t$ shock in each year allows the model to match GDP changes in the data exactly.

[16]The same method is used in Gertler, Kiyotaki and Prestipino (2020). The CBO estimated potential GDP growth from 2008 to 2016 is 1.4%; this estimation is similar to the average growth rate of GDP during this period.

equity of banks adjusted for inflation. All quantity data, including real financial sector equity, real consumption, and real investment, are detrended by the average GDP growth rate from CBO and presented as percentage differences compared to the 2007 level.

Results reveal that, from 2007 to 2009, both the magnitude of the decline in financial sector equity and the spike of the liquidity premium in the model are close to their data counterparts, although the financial sector equity in the model has a smaller drop in 2009.

As shown in the left bottom panel of Figure 6, the model accounts well for the percentage changes in consumption. In the right bottom panel, we see that the model and data paths for output exactly match by construction.

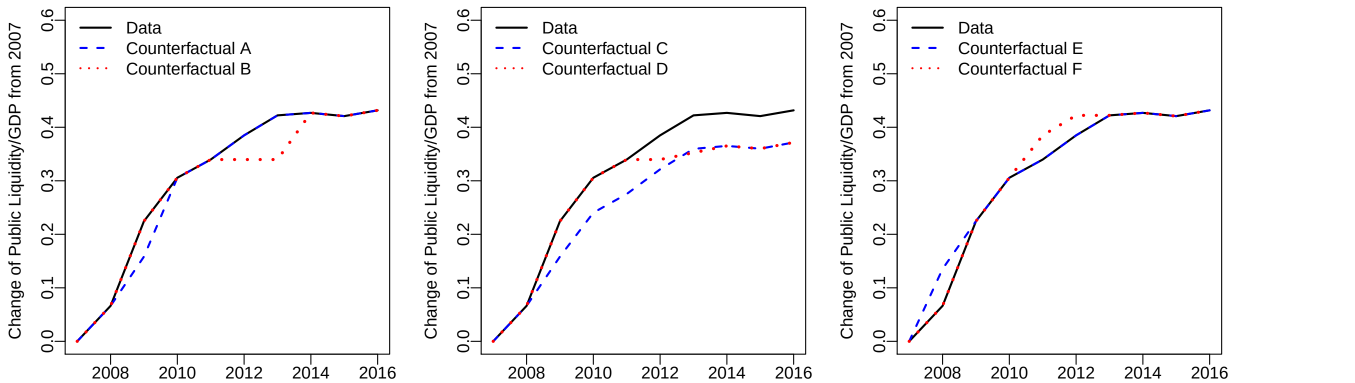
In summary, the model simultaneously matches output, bank equity, consumption, and liquidity premium. These quantities and prices are important for interpreting the importance of public liquidity around the 2008 financial crisis.

## *6.2. The Liquidity Impact of Fed Policies*

The Fed has been praised for its swift actions during the 2008 crisis to avoid an economic meltdown like the great depression. A key part of their policy actions is the quantitative easing policies. The following counterfactual exercise is particularly interesting to understand the importance of Fed policies: If the Fed had delayed its policy responses during the 2008 crisis, how worse would the economy have been?

I will implement counterfactual analyses on public liquidity supply, based on the assumption that shocks $dZ_t$ and $dN_t$ remain the same in the counterfactual world as in the baseline case. I have designed four experiments that change the liquidity path; these experiments are illustrated in Figure 7. In panel (a), the two experiments, A and B, are counterfactuals of delaying QE1 and QE3 by one year. In panel (b), the two experiments, C and D, are counterfactuals of not implementing QE1 or QE3 at all, which amount to a permanent reduction in public liquidity supply. In panel (c), the two experiments, E and F, are counterfactuals of implementing QE1 and QE3 earlier by one year, respectively.

Figure 8 illustrates the differences between the counterfactual and the baseline in all six experiments. Panel (a) shows that if the Fed had delayed QE1 by one year, the output would have been reduced by $-0.17\%$ in 2009, and the effect becomes slightly stronger in the next two years due to nonlinearity in the model. The sum of these differences (cumulative effect) from 2009 to 2015 is about $-0.9\%$. However, if the Fed had not implemented QE1, the output decline would have been much stronger, as reflected by panel (c). Again, due to nonlinear effects, the output difference becomes more significant in 2010 to 2012, with a total cumulative difference of $-1.7\%$ from 2009 to 2015.




(a) Delaying QE1 and QE3 (b) Without QE1 and QE3 (c) Ealier QE1 and QE3

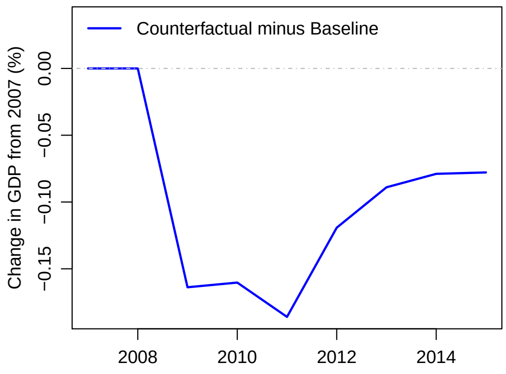
Figure 7. **Counterfactuals of Public Liquidity Supply.** This figure illustrates counterfactuals of public liquidity supply around the 2008 financial crisis. Panel (a) shows counterfactuals A and B that delay the implementation of QE1 and QE3, respectively. Panel (b) shows counterfactuals C and D that do not implement QE1 and QE3, respectively, i.e., a reduction of public liquidity/GDP till 2016. Panel (c) shows counterfactuals E and F that implement QE1 and QE3 earlier by one year, respectively.

In comparison, the later-stage QE3 has a much smaller and more temporary impact. As shown by panel (b) of Figure 8, the output in 2013 would have been reduced by $-0.03\%$ if QE3 had been delayed by about one year, but the impact is quite temporary, because bank equity had largely recovered by 2013. The total impact from 2012 to 2015 is around $-0.06\%$. As shown in panel (d), if QE3 had not been implemented, the cumulative difference from 2012 to 2015 is $-0.07\%$, and negligible after 2015. Thus, the cumulative impact of a persistent QE1 is $1.7/0.07 = 24$ times stronger than a persistent QE3.

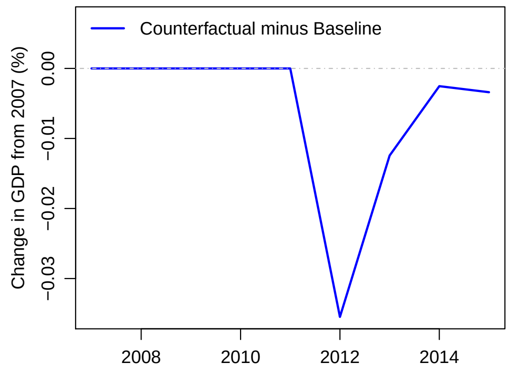
Finally, in counterfactual E of panel (e), we find a very powerful impact of implementing QE1 earlier by one year, where the magnitude of output improvement is 0.6% in 2009, and the cumulative impact from 2008 to 2015 is 4%. In comparison, as shown by panel (f), implementing QE3 earlier by one year has much smaller benefits, and the total cumulative effect is 0.13%.

The above results have two main implications.

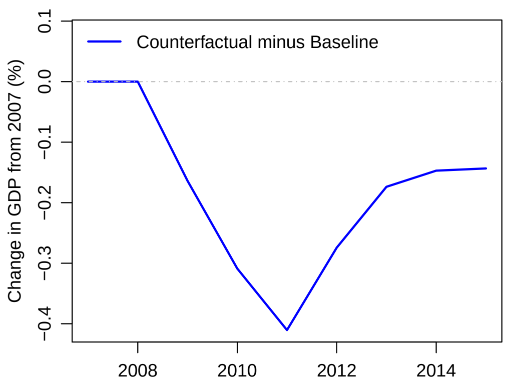
- The effectiveness of QE is highly state-dependent and very sensitive to the timing. Because QE1 is at the time of severe financial distress while QE3 is at the time of recovery, for similar quantity of public liquidity expansions, their cumulative effects differ by about twenty times. Furthermore, advancing each QE implementation makes the policy much more effective.
- Prolonged expansion of public liquidity offers benefits, but the marginal improvement over output declines sharply over time.



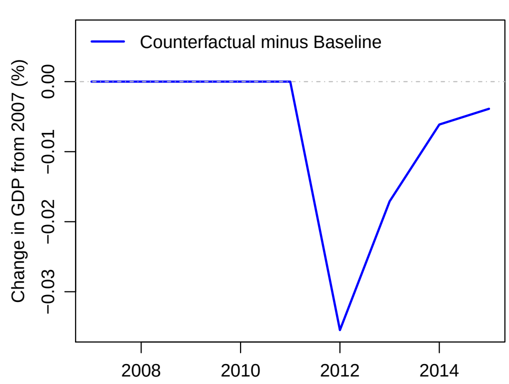

(a) Counterfactual A: Delaying QE1



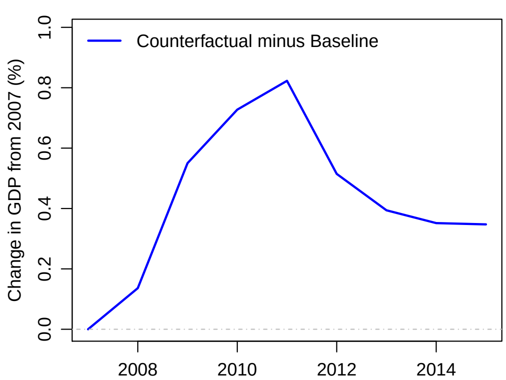

(b) Counterfactual B: Delaying QE3



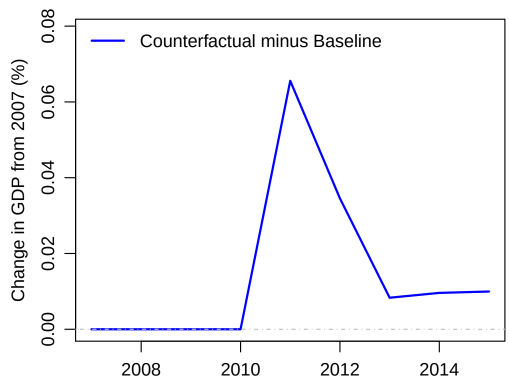

(c) Counterfactual C: Without QE1




(d) Counterfactual D: Without QE3




(e) Counterfactual E: Earlier QE1




(f) Counterfactual F: Earlier QE3

Figure 8. **Counterfactual Results of QE Policies.** The unit is percentage in reference to the 2007 GDP. Thus, −0.1 means that GDP is 0.1 percentage lower in the counterfactual case. Panels a-f correspond to counterfactual scenarios A-F in Figure 7.

## 6.3. *QE Policies and Non-Financial Shocks*

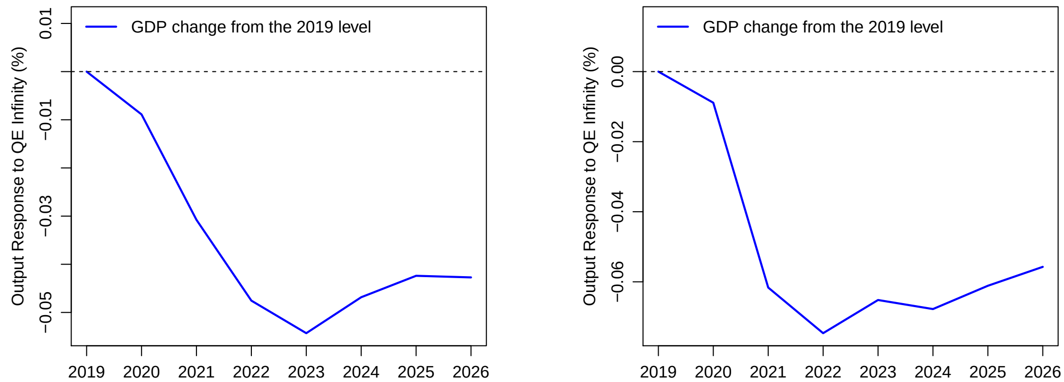
The coronavirus pandemic in 2020 caused government lock-down policies, which are necessary for fighting the pandemic but have severe impact on firms' day-to-day business operations. This pandemic is closer to a real shock rather than a financial shock. In response, on March 15, the Fed announced the purchase of $700 billion worth of Treasurys and MBS. Then after a week, on March 23, the Fed announced the "QE infinity" initiative, with "unlimited purchases" of Treasurys and MBS. Given the other policy responses launched at the same time, such as fiscal stimulus and a reduction of the nominal interest rate, it is difficult to measure the QE policy's effectiveness directly from financial markets.

To illustrate how the channel in this paper works in such an event, I focus on two aspects of the COVID-19 recession – sudden supply-side shocks and a large public liquidity expansion, and then investigate the implications through the lens of the model. I implement the following exercise: I start simulating the model from 2019, with state variables extracted from previous GDP-matching exercises. The "QE infinity" policy is modeled as expanding public liquidity/GDP by 200% in 2020 only. In the first experiment, the Brownian capital shock at 2020 is set so that the model exactly matches the output drop in 2020, and zero otherwise. In the second experiment, I assume that the economy suffers continued capital shocks for two more years until 2022, and the Brownian shock is set to zero otherwise. Then I calculate the output path with versus without QE infinity, so the difference implies the impact of the QE infinity policy.

As shown in Figure 9, counterfactual results are quite surprising: with either short-lived negative shocks (panel a), or prolonged negative-shocks (panel b), the impact of QE infinity on output is negative. The cumulative effect on output is about $-0.27\%$ (panel a) to $-0.38\%$ (panel b).

Why are the effects negative? According to Proposition 2, a larger public liquidity supply increases bank capital holding $x^K$ and the value of capital $q$, thus improving output. However, that also brings more vulnerability to the capital shocks $dZ_t$ due to the larger bank capital holding. In other words, although the increased public liquidity supply reduces bank vulnerability to the liquidity crisis shock $dN_t$, it makes banks more vulnerable to negative capital shocks as in COVID-19.

The model does not incorporate bank regulation, which can curb the vulnerability of banks to real shocks. Considering bank regulation is more fruitful in models that include shadow banks (Huang (2018); Begenau and Landvoigt (2021); Dávila and Walther (2021)) or with belief variation (Krishnamurthy and Li (2020); Maxted (2022)).




(a) The Impact of QE infinity Assuming Only 2020 with Negative Shocks



(b) The Impact of QE Infinity Assuming Prolonged Negative Shocks

Figure 9. **Impact of QE Infinity on Output through the Liquidity Channel.** This figure shows the output response to QE infinity. The counterfactual experiment assumes that QE infinity in 2020 increases public liquidity supply/GDP by 200%, and scales back afterwards. In panel (a), the Brownian shocks in 2020 is set to match the GDP decline of that year, and zero otherwise. In panel (b), the Brownian shocks in 2021 and 2022 are set the same as that in 2020 (i.e., assuming that the pandemic effect persists longer), and zero otherwise.

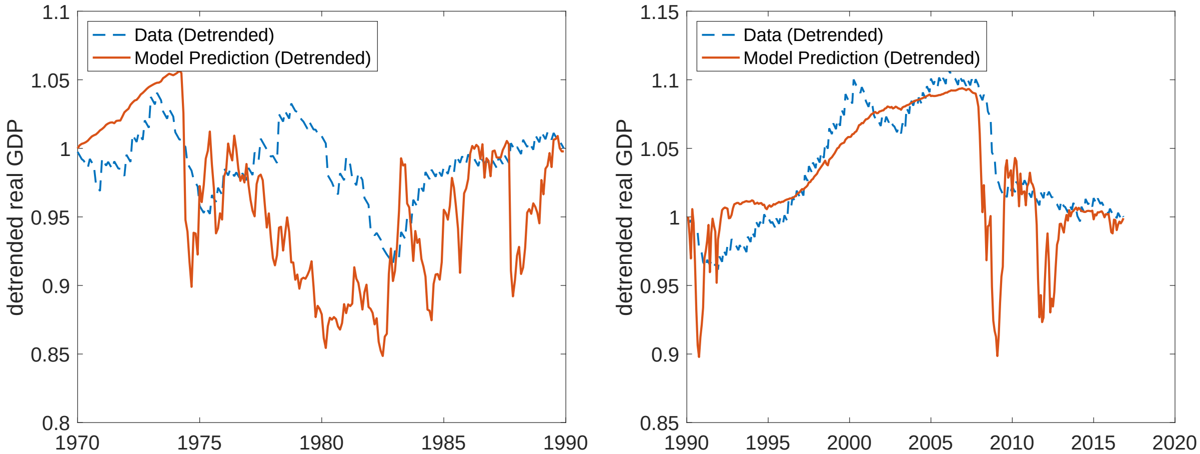
## *6.4. Implications for Other Macroeconomic Dynamics*

In this subsection, I provide two sets of quantitative evaluations of the model. First, I report output dynamics implied by the same exercise of Section 5.1. I find a structural break in 1990, and also the difficulty of simultaneously matching asset-price volatility and macroeconomic volatility. Second, I report various non-targeted unconditional moments from model simulations and contrast them with the data. The model matches major moments well, but falls short in several aspects due to its simplicity. I also discuss how to enrich the model to improve its quantitative performance.

First, the exercise of Section 5.1 generates the time series of state variables. I assume that $dN_t = 1$ in Sep 2008 but $dN_t = 0$ otherwise. Then the productivity shocks $dZ_t$ can be inferred by matching the dynamics of $w_t$. After knowing both the shocks and state variables, I simulate the output path, and then compare the detrended time series with the data counterpart. Given that the model is not designed for understanding long-term growth, the detrending step serves to focuses on output fluctuations.

Results are shown in Figure 10. I find that the model does much better for the post-1990 period, so I separate it with the pre-1990 period. Across both panels, the model-implied output path is more volatile compared to the data counterpart. Since the model matches bank equity ratio, which is driven by bank stock prices, these results echo the literature that asset price volatility is much higher than macroeconomic volatilities. Before 1990, the GDP

Figure 10. Model-Implied Output Process and Data Counterparts.




(a) Pre-1990 (b) Post-1990

*Notes:* This figure shows the model-implied output path and contrast it with the data counterpart. Both paths are detrended by the corresponding average growth rates. The state variable series $(w_t, B_t)$ is calculated by matching the model-implied bank equity ratio and public liquidity/GDP to the data counterparts, in the same way as the liquidity premium exercise in Section 5.1.

path in the data is much smoother than the model-implied path, indicating that banking in the data is less important than in the model for that period.

Nevertheless, the model still captures major variations in the output process, such as the boom and bust around early 1970s, the early 1990s recession (Fed tightening), the boom before 2008, and the great recession in 2008 (global banking crisis). The model implies a recession around 2012 (European debt crisis), but the U.S. output is not affected much, indicating a tight connection between asset markets of the U.S. and the Europe, but much looser connection in the real economy.

To better understand model performance, I report various model moments for the pre-1990 and post-1990 subsamples in Table 3. We find that the model-implied volatilites of output growth, consumption growth, investment growth are not too far from the data. As expected, since the exercises exactly matches bank equity, the volatility of bank equity growth is the same as the data. The model-implied liquidity premium is much less volatile than the data counterpart before 1990, but very close afterwards, consistent with the pattern reflected by Figure 4. This indicates that in reality, apart from bank capitalization, there are other forces not captured by the model that also drive the liquidity premium. We also note that the model implies too low risk-free rate volatility compared to the data counterpart. This is due to the simple log-utility specification. Finally, volatility of $\sigma^K dZ_t$ from the implied $dZ_t$ shocks is about twice that of $vol(\sigma^K dZ_t) = 3\%$ from an unconditional simulation. This is

Table 3: Model Moments from the Asset Pricing Exercise in Section 5.1

| Moments (all units are %) | Pre-1990 | | Post-1990 | |
|---|---|---|---|---|
| | Model | Data | Model | Data |
| vol(liquidity premium) | 0.12 | 0.40 | 0.15 | 0.22 |
| vol(output growth) | 4.93 | 2.60 | 3.77 | 1.73 |
| vol(bank equity growth) | 26.84 | 26.84 | 25.67 | 25.67 |
| vol(consumption growth) | 2.61 | 2.04 | 2.03 | 1.71 |
| vol(investment growth) | 10.20 | 6.49 | 7.84 | 6.27 |
| vol(risk free rate) | 0.53 | 3.23 | 0.51 | 2.45 |
| vol(deposit spread) | 0.43 | 1.10 | 0.24 | 0.84 |
| vol($\sigma^K dZ_t$) | 8.04 | — | 7.70 | — |

*Notes*: this table shows model moments from the asset pricing exercise in Section 5.1, where state variables are inferred from the bank equity ratio and public liquidity/GDP in the data. The main data sample is from 1970 to 2016, but split into pre-1990 and post-1990. All growth variables are calculated as yearly growth and all units are in percentage points. The risk free rate in the data is the federal funds rate, and the deposit spread is from Krishnamurthy and Li (2023). Bank equity is the total equity of banks and broker-dealers from He and Krishnamurthy (2019) starting at 1973.

not contradictory to the model, because the exercise is not reporting unconditional moments from model simulation, but from a stringent exercise that asks for perfectly matching bank equity ratio (and the model does not target at bank equity ratio volatility or bank leverage volatility).

Finally, I simulate the model for 10000 years at monthly frequency, and then compare model-implied second-order moments, including volatilities and correlations, with the data counterparts on a sample that starts from 1920. I report results in Appendix section B.4. Most volatilities are of similar magnitudes between model and data, including the volatilities of output growth, bank equity growth, consumption growth, investment growth, and deposit spread. The model generates lower unconditional volatility of the liquidity premium and the risk-free rate, and these results are due to similar reasons explained in the previous exercise. Furthermore, all the correlations are of the same sign, and most are also of a similar order of magnitude, such as the correlation between consumption growth and output growth. The main gap is on the correlation between change of deposit spread and output growth, likely due to not modeling monetary policy and its endogenous reaction to output gaps.

# 7. Conclusion

In this paper, I study the impact of public liquidity on financial crises via the liquidity insurance channel, where banks demand public liquidity as self insurance against endogenous liquidity problems. The mechanism relies on three frictions: bank equity frictions, runnable bank debt, and illiquidity discounts on productive capital. All of them are necessary to generate a connection between the public liquidity supply and the real economy.

A key feature of the liquidity insurance channel is that the value of liquidity depends on the state of financial fragility. The VAR results are supportive of this channel. Moreover, the calibrated model can explain 40% of the monthly variations in the liquidity premium, which further corroborates the liquidity insurance channel.

The model implies that a larger supply of public liquidity reduces the severity of financial crises, increases bank lending, and improves output. On the other hand, it leads to higher bank leverage and more vulnerability to real shocks, such as COVID-19.

The model replicates well the dynamics of output, bank equity, and the liquidity premium around the 2008 financial crisis. Counterfactual analyses reveal that QE1 significantly improved the output, and the benefits are much larger than QE3 at the later stage, when the financial sector has partially recovered and the economy has a larger public liquidity supply. Since all of these results are only through the liquidity insurance mechanism, a comprehensive evaluation of QE policies requires incorporating other channels.

The model can be extended in different ways to address other questions. One direction is to introduce both monetary and fiscal authorities and study the implementation of liquidity expansions. Another direction is to analyze the impacts of government defaults on the financial sector. Finally, the model can be extended to allow for a less heavily regulated sector, such as shadow banking, to study the dynamics of financial fragility and regulatory responses.

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# Online Appendix to
# "Public Liquidity and Financial Crises".

Wenhao Li[1]

September 13, 2023

## Appendix A. Proofs and Properties

In this section, I will list all the proofs and properties of the model. I start with discussions of modeling assumptions and a summary list of notations.

### *A.1. Discussions of Modeling Assumptions*

*Different Productivities*

The assumption that banker-operated capital has higher productivity captures the downside of a weaker bank balance sheet. This assumption is common in the macro-finance literature, e.g., Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Gertler, Kiyotaki and Prestipino (2020). In a richer model, the bank-held capital can be allowed to have a decreasing return to scale, which implies that the marginal return to the bank-held capital falls below the marginal return to the household-held capital at a certain point. This feature is also present in Kiyotaki and Moore (1997).

An alternative way of modeling is to assume that bankers have lower risk aversion, and therefore, risky capital is more valuable if bankers have higher wealth, as in Drechsler, Savov and Schnabl (2018). The higher value of capital will feed into investment and economic growth. In reality, both features matter: banks indeed provide a lending service that is not directly replaceable by households (Schwert, 2018), and, in general, financial institutions are less risk-averse than households, as is reflected in much higher leverage and risk-taking. For simplicity, the model captures the first feature. Introducing the second feature can potentially quantitatively improve the performance of the model, but it requires general forms of utility functions and is technically more challenging.

[1]Marshall School of Business, University of Southern California. Email: liwenhao@marshall.usc.edu.

*Financial frictions*

The first financial friction is the equity issuance friction. The assumption of no equity issuance by banks in this paper is standard in the literature (Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2015). This friction can be microfounded as a limit case of an agency friction in which bankers can privately divert resources at the cost of depositors, as in He and Krishnamurthy (2011), Di Tella (2017, 2019). Moreover, the equity constraint makes the financial market incomplete. As a result, bank-owned capital and household-owned capital are two different types of segmented asset. An implicit assumption is that banks and households cannot write contracts on aggregate risks. Indeed, if we allow agents to write contracts on the aggregate state, the balance sheet channel is eliminated, as in Di Tella (2017).

The second financial friction is the bank-run incentive. In this model, the financial panic is due to the fear of bankruptcy that is driven by real shocks. Thus, the modeling approach is different from the sunspot equilibria in the literature (Diamond and Dybvig, 1983; Postlewaite and Vives, 1987; Peck and Shell, 2003), and provides a highly tractable alternative to model liquidity disruptions in the financial sector.

The third friction is the fire sale price discount, which is just a wealth transfer from sellers to buyers. I have assumed that the fire sale price is lower than the post fire sale equilibrium price, which can be microfounded by certain market frictions, such as search frictions (Lagos and Rocheteau, 2009) and slow-moving capital (Duffie, 2010). In the 2008 financial crisis, a temporary price discount was indeed quite prevalent during systemic fire sales, as documented by Stanton and Wallace (2011) and Merrill et al. (2013). The temporary discount is a salient feature of fire sales (Shleifer and Vishny, 2011).

Another interpretation for $\alpha^0$ is a cost of obtaining emergency funding. Even if there is no actual fire sale, as long as banks have difficulty obtaining emergency financing without substantial costs, banks suffer losses and trigger a vicious cycle in capital price decline and bank equity drops.

*Government*

The government is modeled in a highly stylized way, with wealth taxation and simple rules of government spending. In this model, government bonds can be interpreted as the liabilities of a combined central bank and the government. Taxation provides government the flexibility to adjust the quantity of government bonds without distorting individual decisions. The model can be readily extended to allow for distortionary taxation such as capital income tax, wealth tax, and other types of distortions.

### *A.2. Summary of Notations*

I summarize all notations below.

1. Aggregate quantities:
   - $K_t$: aggregate amount of capital in the economy.
   - $W_t^b$: aggregate banker wealth.
   - $W_t^h$: aggregate household wealth.
   - $Y_t$: aggregate output.
   - $A_t K_t$: total government holding of non-deposit bank debt.
   - $\mathcal{T}_t$: total taxation.
   - $\psi_t$: share of capital owned by bankers.
   - $g_t K_t$: total government spending.
2. State variables
   - $K_t$: total amount of capital.
   - $w_t$: the fraction of banker wealth among total wealth.
   - $B_t K_t$: total amount of government bonds.
3. Aggregate shocks:
   - $dZ_t$: the Brownian shock that affects capital dynamics.
   - $dN_t$: the liquidity crisis shock.
4. Endogenous asset prices and returns:
   - $q_t$: the value for each unit of capital.
   - $d\bar{R}_{j,t}^K$: return of capital held by bankers.
   - $d\underline{R}_{j,t}^K$: return of capital held by households.
   - $dR_t^B = r_t^B dt$: the return on government bonds.
   - $dR_t^D = r_t^D dt$: the return on deposits.
   - $dR_t^{ND} = r_t^{ND} dt - \kappa_{t-}^{ND} dN_t$: the return on non-deposits, which has exposure to the liquidity crisis shock $dN_t$.
   - $\mu_t^R$: the non-dividend component of capital return.
5. Drifts:
   - $\mu_t^q$: the drift of capital price dynamics $dq_t/q_t$.
6. Volatilities:
   - $\sigma_t^q$: the volatility of capital price dynamics $dq_t/q_t$.
7. Jumps:

- $\kappa_t^q$: the loss of capital value in a crisis at time $t$.
- $\kappa_t^{ND}$: the loss of non-deposit value in a crisis at time $t$.
- $\kappa_t^{fs}$: firesale benefits for households that purchase capital at a low price in a crisis at time $t$.

8. Individual choices.

- $k_{j,t}$: unit of capital held by individual $j$.
- $c_{j,t}^b$: banker consumption.
- $c_{j,t}^h$: household consumption.
- $\hat{c}_{j,t}^b$: banker consumption per unit wealth.
- $\hat{c}_{j,t}^h$: household consumption per unit of wealth.
- $x_{j,t}^K$: banker capital holding per unit of wealth.
- $y_{j,t}^K$: household capital holding per unit of wealth.
- $x_{j,t}^B$: banker government bond holding per unit of wealth.
- $y_{j,t}^B$: household government bond holding per unit of wealth.
- $x_{j,t}^D$: banker total deposit issuance per unit of wealth.
- $y_{j,t}^D$: household deposit holding per unit of wealth.
- $x_{j,t}^{ND}$: banker non-deposit funding per unit of wealth.
- $y_{j,t}^{ND}$: household investment in non-deposits per unit of wealth.

## *A.3. Property of the Investment Function $i(q)$*

The function $i(q)$ is solved from

$$\phi'(\mu^K) = q$$

Because $q \geq 0$, and the range of $\phi'(\mu^K)$ includes $\mathbb{R}^+$ from properties of $\phi(\cdot)$ in (2), we always have a solution from the above equation. Since $\phi'(\cdot)$ is a strictly increasing function, the solution is unique, and the unique growth rate of capital $\mu^K$ is an increasing function of capital price $q$.

Since capital price $q > 0$, we obtain $\phi'(\mu^K) > 0$. Therefore, the investment function $i(q) = \phi(\mu^K(q))$ increases in $\mu^K$, which increases in $q$. The above implies that $i(q)$ is an increasing function of $q$.

## *A.4. Proof of Lemma 1*

*Proof.* By model assumption, when a crisis shock $dN_t$ occurs, the capital holding among a $\theta$ fraction of banks is completely destroyed. The government takes over the banks, pays the insured depositors in full and bankers a small reservation value ($\varepsilon$ fraction of pre-shock equity), and all other creditors obtain zero recovery in their debt.

Among non-deposits, a maximum fraction $\beta > 0$ is allowed to be withdrawn in a crisis while the rest $1-\beta$ stays with banks, which reflects the reality that banks issue longer-term debt that is not susceptible to short-term liquidity problems. In what follows, I prove that for non-deposits, households withdraw the maximum fraction $\beta$ in equilibrium.

For an active non-deposit (i.e., immediately withdraw funding is allowed in a crisis), if the decision is to withdraw, the recovery is full value regardless, but it forgoes the interest payment in the $dt$ interval by $r^{ND}dt$. If the decision is to stay with the bank, then with probability $1-\theta$, not only the principal amount is recovered, but also the $r^{ND}dt$ amount of interest payment is earned. However, with probability $\theta$, the whole face value is lost.

Consequently, the difference of payoff between staying with the bank versus withdrawing the funding is $(1-\theta)r^{ND}dt - \theta$, which is negative regardless of $r^{ND}$ and $\theta$, because $\theta$ is of order 1 while $r^{ND}dt$ is of order $dt$. Thus, households choose to withdraw all of their active credit to banks (fraction $\beta$ of all non-deposit lending to banks) and suffer losses for the rest $1-\beta$, resulting in the aggregate non-deposit return of

$$r^{ND}dt - \theta(1-\beta)dN_t$$

In normal times, asset markets are liquid by assumption. Therefore, whenever households withdraw funding from banks, banks can sell assets without the fire-sale discount, thus incurring zero loss. More importantly, because in normal times there is no capital destruction, bank funding market does not freeze. Whenever households run on banks, they can also immediately borrow from other households via non-deposits, which eliminates any sales of assets. In summary, banks suffer no losses if households withdraw funding in normal times, and all creditors are paid off in full. Therefore, households obtain no benefit by running on banks in normal times. □

## *A.5. HJB Equations*

According to household's budget constraint, the scaling property of banker wealth remains, i.e. if we scale household's wealth by a factor of $\bar{\alpha}$, then by following the same strategy (consumption strategy is the consumption/wealth ratio), the new wealth at each time $t$ will be just $\bar{w}^h_{j,t} = \bar{\alpha} w^h_{j,t}$. Regardless of the starting wealth, the portfolio choices and consumption/wealth ratio should be the same, because otherwise, we can use the scaling property to arrive at a contradiction. Denote $\bar{c}^h_{j,t}$ and $\bar{y}^D_{j,t}$ as the optimal consumption and portfolio choice in insured deposits, given an initial wealth of 1. By definition, the paths of $\bar{c}^h_{j,t}$ and $\bar{y}^D_{j,t}$ are not depending on $w_{j,h}$ but only on the aggregate state variables $(w, B)$. Because of the scaling property, for any different initial wealth $w^h_j$, the optimal consumption at any time $t$ is $\bar{c}^h_{j,t} w^h_j$ and the optimal total deposit holding is $\bar{y}^D_{j,t} w^h_j$.

Consequently, we obtain

$$\begin{aligned} V^h(w, B, w^h_j) &= E\left[\int_0^\infty e^{-\rho t} \log(w^h_j \bar{c}^h_{j,t}) dt | w_0 = w, B_0 = B, w^h_{j,0} = w^h_j\right] \\ &= E\left[\int_0^\infty e^{-\rho t} \left(\log(w^h_j) + \log(\bar{c}^h_{j,t})\right) dt | w_0 = w, B_0 = B, w^h_{j,0} = w^h_j\right] \\ &= \frac{1}{\rho} \log(w^h_j) + v^h(w, B) \end{aligned} \tag{A1}$$

where $v^h(w, B)$ is a function that only depends on the aggregate state of the economy.

With the log-form of the value function, we use $c^h_j$ to denote the current household consumption and $y^D_j$ to denote the fraction of wealth currently invested in deposits. Then the HJB equation becomes

$$V^h(w, B, w^h_j) = \max\left\{\log(c^h_j)dt + (1 - \rho dt)E\left[V(w, B, w^h_j) + \frac{1}{\rho} \cdot d\log(w^h_j) + dv^h(w, B)\right]\right\}$$

The greatest simplification comes from the property that individual state $w^h_j$ is separable from the aggregate state $(w, B)$, so the portfolio decisions are directly solvable. Rearranging terms, we obtain

$$\rho V^h(w, B, w^h_j) = \max\left\{\begin{array}{c} \rho\log(c^h_j) + \mu^h_j - \frac{1}{2}(\sigma^h_j)^2 \\ +\lambda\left(\log(w^h_j + \Delta w^h_j) - \log(w^h_j)\right)) + \cdots \end{array}\right\} \tag{A2}$$

where we denote $\mu^h_j$ as the drift of $dw^h_j/w^h_j$, $\sigma^h_j$ as the volatility of $dw^h_j/w^h_j$, and $\Delta w^h_j$ as the jump of wealth during the crisis shock. At the end of (A2), we omit the terms related to aggregate states $w$ and $B$ because they do not involve individual household portfolio and consumption choices.

Therefore, we can plug in the dynamics of $dw^h_j/w^h_j$ as in (13), and obtain the equivalent optimization problem as in (28). The verification step for the HJB equation is standard in the literature and thus omitted.

For banks, there is a transition to households at the rate of $\eta$. Therefore, the bank value function is

$$V^b(w, B, w^b_j) = E[\int_0^T e^{-\rho t} \log(w^b_j \bar{c}^b_{j,t})dt + e^{-\rho T} V^h(w_T, B_T, w^h_{j,T}) | w_0 = w, B_0 = B, w^h_{j,0} = w^h_j]$$

where $\bar{c}^b_{j,t}$ is defined in the same way as $\bar{c}^h_{j,t}$, representing the optimal consumption when the initial wealth is one unit. The HJB equation is

$$V^b(w,B,w^b_j) = (1-\lambda dt)\cdot\big(\log(\bar{c}^b_j)dt + \log(w^b_j)dt + (1-\rho dt)(V^b(w,B,w^b_j) + E[dV^b(w,B,w^b_j)])\big) + \lambda dt \cdot V^h(w,B,w^b_j)$$

Plugging in the functional form of $V^h$ in (A1), we get

$$(\rho+\lambda)V^b(w,B,w^b_j) = (1+\frac{\lambda}{\rho})\log(w^b_j) + \cdots$$

where we omit other terms. We find that the following functional form of $V^b$ is consistent with the HJB equation:

$$V^b(w,B,w^b_j) = \frac{1}{\rho}\log(w^b_j) + v^b(w,B)$$

With this value function, following similar derivations as in (A2), we obtain the equivalent optimization problem as in (30). Again, the verification step for the HJB equation is standard in the literature and thus omitted.

## A.6. First-Order Conditions

First, we solve for household benefit from fire sales $\kappa^{\text{fs}}$ from the following equality:

$$\underbrace{(1-w)}_{\substack{\text{total household}\\ \text{wealth}}} \cdot \kappa^{\text{fs}} = \underbrace{w}_{\substack{\text{total banker}\\ \text{wealth}}} \cdot \underbrace{\frac{\Delta x}{(1-\alpha^0)q}}_{\substack{\text{fire sale quantity for each}\\ \text{unit of banker wealth}}} \cdot \underbrace{\alpha^0 q}_{\substack{\text{wealth transfer for}\\ \text{each unit sale}}}$$

$$\Rightarrow \kappa^{\text{fs}} = \frac{\alpha^0}{1-\alpha^0}\Delta x \frac{w}{1-w} \tag{A3}$$

Then we derive the first-order conditions according to the simplified problems as in (28) and (30).

For banks, the first-order condition for capital holding is

$$\mu^R + \frac{\bar{A}}{q} - r^{ND} = x^K(\sigma^q + \sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta\cdot 1_{\Delta x>0}}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} \tag{A4}$$

which suggests that the excess return earned by capital is due to both its volatility and compensation for its losses in a crisis, including both the decline of price $\kappa^q$, and the liquidation losses related to $\alpha^0$.

The first-order condition for insure deposits supply $x^D$ is

$$r^{ND} - r^D = c'_D(x^D) - \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{A5}$$

which suggests that the deposit spread, $r^{ND} - r^D$, is driven mainly by two forces: 1) the marginal cost of producing deposits, $c'_D(x^D)$; and 2) the benefit of having extra deposits on reducing non-deposit funding. The former force makes deposit production more costly to banks and thus banks pay lower deposit rate. The latter increases banks' willingness to pay for deposits and thus increases the deposit rate. When the deposit production cost dominates, we obtain a positive deposit spread.

The first-order condition for holding government bonds $x^B$ is

$$r^{ND} - r^B \geq \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{A6}$$

where the equality holds if $x^B > 0$. We note from the right-hand side that government bonds is priced at lower yields because it provides a hedge to capital liquidation.

For households, the first-order condition for capital holding is

$$\mu^R + \frac{A}{q} - r^{ND} \leq y^K(\sigma^q + \sigma^K)^2 + \lambda\theta\frac{1-\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^q - \kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}}$$

where the equality holds if $y^K > 0$. The above right-hand side composes three terms: 1) volatility risk; 2) compensation for the capital destruction (with probability $\theta$ in a crisis); 3) compensation for the price drop of capital (if capital is not directly destroyed). Since $\theta$ is quite small, quantitatively only terms 1) and 3) matter for the household pricing of capital.

The first-order condition over deposits holding $y^D$ is

$$r^{ND} - r^D = v'_D(y^D) + \lambda\theta\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}} \tag{A7}$$

which implies that the deposit spread $r^{ND} - r^D$ should always be positive, for two reasons: 1) households have extra liquidity value when they hold deposits, reflected by $\beta_d\rho/y^D$; 2) non-deposits have a small amount of credit risk. Since $\kappa^{ND} = \theta(1-\beta)$ is a very small number, the main driving force is the household special demand for liquidity. Taking (A5) and (A7) together, we find that a larger household demand for deposits increases the deposit spread, which requires the banking sector to accommodate with higher $x^D$ and thus larger marginal cost of deposit production, justifying the higher deposit spread in the first place.

## A.7. *Proof of Proposition 1 and 2*

The proofs of Proposition 1 and 2 are combined since they are both under the same assumptions and results are closely interlinked.

First, I show that $x^K$ and capital value $q$ are positively correlated, given asset price dynamics. Plugging the definition of $\psi_t$ in (17), the wealth identity (23), and the consumption rule (31) into the resource constraint (26), we get

$$wx^K(\bar{A} - \underline{A}) = \rho q + \frac{q}{q + B_0}\left(i(q) + g - \underline{A}\right), \tag{A8}$$

where $i(q) = \phi(\mu^K(q))$ is the optimal investment as a function of $q$, which is an increasing function of $q$. We find that the right hand side of (A8) increases in $q$, which implies that in equilibrium, a higher bank capital holding $x^K$ is associated with more valuable capital. Let this relationship be $q(x^K)$.

Denote

$$g(x^K, \Delta x) = x^K(\sigma^q + \sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} - \left(\mu^R + \frac{\bar{A}}{q(x^K)} - r^{ND}\right) \tag{A9}$$

which clearly is an increasing function in $x^K$, but a decreasing function over $\Delta x$. Since the first-order condition in (A4) is equivalent to $g(x^K, \Delta x) = 0$, we find that $\Delta x$ and $x^K$ are negatively related. The idea is that given asset price dynamics, to justify banks holding capital, more vulnerability has to be compensated with higher bank leverage and thus more profits. Furthermore, we also find that the total crisis destruction, $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$, as a whole according to (A9) decreases with $x^K$, and therefore increases with $\Delta x$.

Second, I show that financial fragility $\Delta x$ is lower when there is a larger public liquidity supply $B$. I will prove by contradiction. Suppose that $\Delta x$ becomes lower, which then implies lower $x^K$ and larger $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$. According to equation (A11), bank deposit production $x^D$ expands. However, these changes are contradictory to the larger $\Delta x$, because by definition,

$$\Delta x = \beta(x^K - x^D - 1) - (1-\beta)x^B \tag{A10}$$

When we simultaneously have higher $x^B$, lower $x^K$, and larger $x^D$, equation (A10) implies a lower $\Delta x$. Contradiction! As a result, a larger public liquidity supply $B$ reduces financial fragility $\Delta x$, which implies a smaller total crisis destruction, $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$. The first-order condition $g(x^K, \Delta x) = 0$ also implies that larger $B$ increases bank lending $x^K$, which boosts the capital value $q$.

Third, the total productivity of the economy is

$$\left(\psi\bar{A} + (1-\psi)\underline{A}\right) = \rho(q + B_0) + \phi(\mu^K) + g$$

which increases in $q$ (note that $\mu^K$ is an increasing function of $q$). Therefore, the total productivity of the economy increases with public liquidity supply $B$. This finishes the proof for Proposition 2.

Fourth, we show that deposits are crowded out. Equating (A5) and (A7), we obtain

$$c'_D(x^D) - \beta_d \frac{\rho}{y^D} = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} 1_{\Delta x>0} \cdots \tag{A11}$$

where we have omitted the $\kappa^{ND}$ terms since we take the stance of small direct capital destruction. Due to convexity, the function $c'_D(x^D)$ increases in $x^D$. Furthermore, the market clearing condition in (25) implies $y^D = \frac{w}{1-w}x^D$ and thus the left-hand side of (A11) increases in $x^D$. Therefore, we only need to prove that a larger $B$ reduces the right-hand side of (A11). We have already proved that a larger $B$ increases $x^K$. According to the first-order condition in (A4), which is equivalent to $g(x^K, \Delta x) = 0$, the right-hand side of (A11) is smaller with a larger $x^K$., which implies that larger $B$ reduces the right-hand side of (A11). Consequently, a larger $B$ decreases deposits $x^D$.

Fifth, since larger $B$ simultaneously reduce $\Delta x$ but increases $x^K$, according to (A10), we must have at least $x^D$ increasing or $x^B$ increasing. Since we have already proved that $x^D$ decreases with $B$, equation (A10) implies that $x^B$ increases with public liquidity $B$. This finishes the proof for Proposition 1.

Finally, since both $x^B$ and $x^K$ increases with public liquidity supply $B$, and bank leverage is $x^K + x^B$, we find that bank leverage increases with public liquidity supply $B$. This concludes the proof for Proposition 1.

## A.8. *Proof of Proposition 3*

Consider the hypothetical asset where only a portion $(1-\pi) \in (0,1)$ can be redeemed for liquidity in a crisis, while the value of this asset remains the same in a crisis. According to the bank pricing kernel, the spread between $r^{ND}$ and the yield $r^f$ of this asset is

$$r^{ND} - r^f = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\pi-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} 1_{\Delta x>0}$$

Combining the above equation with (A6), we obtain

$$\ell \equiv r^f - r^B = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\pi}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0}$$

Given asset price dynamics, according to the proof for Proposition 1 and 2, we know that the term $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$ decreases in $B$, and therefore, the liquidity premium $\ell$ is smaller when there is a larger public liquidity supply.

## *A.9. Proof of Proposition 4*

*Proof.* As a starting point, the mechanism of public liquidity supply affecting the economy starts from its impact on bank holding of public liquidity. The change of bank portfolio holding then affects the severity of a crisis, bank capital holding $x^K$, and the equilibrium value of capital $q$.

As the structure of the proposition, I will prove the proposition in three parts.

**Case 1:** banks can issue equity.

In this scenario, banks can share the risks with households through issuing equity, and households can also earn a higher return on productive capital by holding bank equity. In a benchmark setting where Modigliani–Miller holds, whether banks issue equity or debt does not matter. However, in the model, issuing equity is superior to issuing debt, because bank debt is associated with bank run risks. Furthermore, from the households perspective, investing in bank equity is superior to directly investing in productive capital, because bank-operated capital yields higher returns. Consequently, banks only issue equity, which allows perfect risk sharing between bankers and households.

Since there is no bank debt, bank runs and fire sales are all eliminated by the equity issuance. Consequently, the liquidity premium is zero. The Ricardian equivalence holds, and the amount of government bonds has no impact on the real economy.

**Case 2:** all deposits are sticky with $\beta = 0$.

When all deposits are sticky, we have the net funding withdrawal

$$\Delta x = (\beta x^{ND} - x^B)^+ = 0$$

As a result, bank holding of government bonds does not affect its losses in a crisis. In the first-order conditions of $x^K$ and $x^D$, the supply of government bonds completely drops off.

Next, I discuss whether the bank run losses during a crisis are still related to the amount of public liquidity supply. Banker wealth jump during a crisis is

$$\kappa^b = (1-\theta)\left(x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x\right) + \theta(1-\varepsilon)$$

Given $\beta = 0$, we have

$$\kappa^b = (1-\theta)x^K\kappa^q + \theta(1-\varepsilon)$$

and

$$\kappa^h = \theta y^K + (1-\theta)y^K\kappa^q + y^{ND}\kappa^{ND}$$

where the fires-sale benefit is zero since there are no actual fire sales when $\beta = 0$. As a result, the changes of banker and household wealth are not related to public liquidity holding, which implies that it does not affect crisis severity.

In summary,, liquidity supply has zero impact on the disruptions of a crisis shock when $\beta = 0$. Neither does it affect the capital price and banks' portfolio choices except for government bonds.

**Case 3:** no fire sale market pressure, or $\alpha^0 = 0$.

The proof for this case is similar to Case 2, since $\alpha^0 = 0$ leads to no fire-sale loss at all and no role of holding public liquidity for banks. □

## *A.10. Evolutions of State Variables*

I will derive the dynamic evolutions of the aggregate state variables $w$ and $K$. Define the evolution of state variable $w$ as

$$dw_t \triangleq \mu_t^w dt + \sigma_t^w dZ_t - \kappa_{t-}^w dN_t \tag{A12}$$

I will derive the explicit expressions for $dw_t$ in two steps. First, with Ito's formula, the dynamics of $W_t^b/W_t^h$ is

$$d\left(\frac{W_t^b}{W_t^h}\right) = \frac{W_{t-}^b}{W_{t-}^h}\begin{pmatrix} (\mu_t^b - \mu_t^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - \eta(1+\frac{W_t^b}{W_t^h}))dt \\ +(\sigma_t^b - \sigma_t^h)dZ_t + \left(\frac{1-\kappa_{t-}^b}{1-\kappa_{t-}^h} - 1\right)dN_t \end{pmatrix}$$

Second, from

$$w_t = \frac{W_t^b}{W_t^b + W_t^h} = 1 - \frac{1}{W_t^b/W_t^h + 1}$$

and Ito's lemma, the dynamics of wealth $w_t$ is

$$dw_t = \frac{\frac{W^b_{t-}}{W^h_{t-}}\left((\mu^b_t - \mu^h_t + (\sigma^h_t)^2 - \sigma^b_t\sigma^h_t - \eta(1+\frac{W^b_t}{W^h_t}))dt + (\sigma^b_t - \sigma^h_t)dZ_t\right)}{\left(W^b_t/W^h_t + 1\right)^2}$$

$$-\frac{\left(\frac{W^b_t}{W^h_t}\left(\sigma^b_t - \sigma^h_t\right)\right)^2 dt}{\left(W^b_t/W^h_t + 1\right)^3} + \left(\frac{1}{1+\frac{W^b_{t-}}{W^h_{t-}}} - \frac{1}{1+\frac{W^b_{t-}}{W^h_{t-}}\frac{1-\kappa^b_{t-}}{1-\kappa^h_{t-}}}\right)dN_t$$

Based on the connection between individual wealth dynamics and aggregate dynamics in (19) and (20), we can express dynamics of $w_t$ as

$$\begin{aligned} dw_t = {} & w_{t-}(1-w_{t-})\left(\mu^b_t - \mu^h_t + (\sigma^h_t)^2 - \sigma^b_t\sigma^h_t - w_t(\sigma^b_t - \sigma^h_t)^2 - \eta\frac{1}{1-w_t}\right)dt \\ & + w_{t-}(1-w_{t-})(\sigma^b_t - \sigma^h_t)dZ_t + w_{t-}(1-w_{t-})\left(\frac{\frac{1-\kappa^b_{t-}}{1-\kappa^h_{t-}} - 1}{1 + w_{t-}(\frac{1-\kappa^b_{t-}}{1-\kappa^h_{t-}} - 1)}\right)dN_t \end{aligned} \tag{A13}$$

With wealth share dynamics expressed, we can write the equilibrium price fixed point equation as

$$\kappa^q = \frac{q(w,B) - q(w\frac{1-\kappa^b}{1-\kappa^h - w(\kappa^b - \kappa^h)}, B)}{q(w,B)} \tag{A14}$$

Next, we denote the aggregate capital process as

$$\frac{dK_t}{K_{t-}} = (\mu^K_t - \delta)dt + \sigma^K dZ_t - \theta dN_t \tag{A15}$$

# Appendix B. Numerical Methods and Model Moments

To solve the model, I first list the equilibrium equation system. Then I present the numerical algorithm to solve the whole model. Finally, I present a summary of model moments and contrast them with data.

## *B.1. System of Equations for Solving the Model*

In equilibrium, we mainly have the following three types of equations: (1) Equilibrium conditions, such as capital market clearing. (2) Individual optimality, such as the optimality of public liquidity and bank debt holding. (3) Definitions, such as taking Ito's formula on price function $q(w, B)$ to get an expression of $\mu^q$.

All of the portfolio choices, $x^K$, $x^B$, etc, are functions of the states $(w, B)$, but for simplicity, we omit the explicit dependence in the expressions below.

1. Market clearing conditions for capital, government debt, and insured deposits:

$$wx^K + (1-w)y^K = \frac{q}{q+B_0} \tag{A16}$$

$$wx^B + (1-w)y^B = \frac{B}{q+B_0} \tag{A17}$$

$$wx^D = (1-w)y^D \tag{A18}$$

By assumption, a fraction $\delta_B$ of government debt is held by banks,

$$wx^B = \delta_B \frac{B}{q+B_0} \tag{A19}$$

The fraction of capital held by banks is

$$\psi = \frac{wx^K}{q/(q+B_0)} \tag{A20}$$

2. Consumption good clearing:

$$\left(\psi\bar{A} + (1-\psi)\underline{A}\right) = \rho(q+B_0) + \phi(\mu^K) + g \tag{A21}$$

where

$$q = \phi'(\mu^K) \tag{A22}$$

3. First-order conditions over capital holdings,

$$\mu^R + \frac{\bar{A}}{q} - r^{ND} = x^K(\sigma^q + \sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta \cdot 1_{\Delta x>0}}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} \tag{A23}$$

$$\begin{aligned} \mu^R + \frac{\underline{A}}{q} - r^{ND} \leq\ & y^K(\sigma^q + \sigma^K)^2 + \lambda\theta\frac{1-\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} \\ & + \lambda(1-\theta)\frac{\kappa^q - \kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}} \end{aligned} \tag{A24}$$

4. First-order condition over government bond holding,

$$r^{ND} - r^B = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} 1_{\Delta x>0} \tag{A25}$$

where the equality comes from the assumption that banks always hold $\delta_B > 0$ fraction of government bonds. Households are assumed to passively hold government debt and therefore they do not have a pricing equation.

5. First-order conditions over insured deposits:

$$r^{ND} - r^D = c_D'(x^D) - \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x > 0} \tag{A26}$$

$$r^{ND} - r^D = v_D'(y^D) + \lambda\theta\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}} \tag{A27}$$

6. Volatilities of capital growth $\sigma^q$, state variable $w$, household wealth growth $\sigma^h$, and banker wealth growth $\sigma^b$:

$$\begin{cases} \sigma^q = q_w w(1-w)(\sigma^b - \sigma^h) \\ \sigma^w = w(1-w)(\sigma^b - \sigma^h) \\ \sigma^h = y^K(\sigma^K + \sigma^q) \\ \sigma^b = x^K(\sigma^K + \sigma^q) \end{cases} \tag{A28}$$

7. Drifts of capital growth $\mu^q$, state variable $w$, household wealth growth $\mu^h$, and banker wealth growth $\mu^b$:

$$\mu^q = q_w'\mu^w + \frac{1}{2}q_{ww}''\left(w(1-w)(\sigma^b - \sigma^h)\right)^2 + q_B'\mu^B \tag{A29}$$

$$\mu^w = w(1-w)\left(\mu^b - \mu^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - w_t(\sigma_t^b - \sigma_t^h)^2\right) - w\eta \tag{A30}$$

$$\mu^h = y^K(\mu^R + \frac{\underline{A}}{q} - r^{ND}) + y^B(r^B - r^{ND}) + y^D(r^D - r^{ND}) + r^{ND} - \rho \tag{A31}$$

$$\mu^b = x^K(\mu^R + \frac{\bar{A}}{q} - r^{ND}) + x^B(r^B - r^{ND}) - x^D(r^D + c_D(x^D) - r^{ND}) + r^{ND} - \rho \tag{A32}$$

8. Jump of banker wealth is $\kappa^b$ (drop in a crisis as a fraction of pre-crisis wealth),

$$\kappa^b = (1-\theta)\left(x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x\right) + \theta(1-\varepsilon) \tag{A33}$$

and household wealth decline is $\kappa^h$

$$\kappa^h = \left(\theta y^K + (1-\theta)y^K\kappa^q\right) + y^{ND}\kappa^{ND} - \kappa^{fs} \tag{A34}$$

with

$$\kappa^{ND} = \theta(1-\beta) \tag{A35}$$

$$\kappa^{fs} = \frac{\alpha^0}{1-\alpha^0}\Delta x \cdot \frac{w}{1-w} \tag{A36}$$

Finally, the equilibrium fixed-point condition for capital value $q$ is

$$\kappa^q = 1 - q(w\frac{1-\kappa^b}{1-\kappa^h - w(\kappa^b - \kappa^h)}, B + \kappa^B)/q(w, B) \tag{A37}$$

9. Balance sheet identities:

$$x^K + x^B = 1 + x^D + x^{ND} \tag{A38}$$

$$y^K + y^B + y^D + y^{ND} = 1 \tag{A39}$$

## *B.2. Algorithm*

Due to jumps in both state variables $w_t$ and $B_t$, the equilibrium system cannot be translated into a standard partial differential equation. If the model has a single state variable $w_t$, then the system can be cast as a delayed ODE. A good reference is Huang (2018). Huang (2018) also features endogenous jumps in $w_t$ and is also built on Brunnermeier and Sannikov (2014). Nevertheless, the problem in my paper involves delayed partial differential equations. I have designed a specific functional iteration algorithm to solve the model.

The structure of the algorithm is as follows.

1. Start with an initial function $q_{(0)}(w, B)$. The initial price function is constructed by interpolating solutions that take $B$ as a constant.
2. For $n = 0, 1, \cdots$. With the given price function $q_{(n)}(\cdot)$, I can solve a new $x^K_{(n+1)}(\cdot)$ and a new jump function $\kappa^q_{(n+1)}(\cdot)$, where in the jump equation (A37), the $q$ function is taken as the last round value $q_{(n)}(\cdot)$. The progressive property of the algorithm guarantees the existence of solutions in each step. Then we can calculate

$$\psi_{(n+1)} = \frac{wx^K_{(n+1)}}{wx^K_{(n+1)} + (1-w)y^K_{(n+1)}}$$

   Finally, I solve the next-round capital value $q_{(n+1)}$ from the resource constraint:

$$\psi_{(n+1)}\bar{A} + (1-\psi_{(n+1)})\underline{A} = \rho q_{(n+1)} + \phi(\mu^K(q_{(n+1)})) + g$$

3. Update the new rounds of price function and the liquidity wealth function with a learning rate $\varpi \in (0, 1)$, so that the final updating is

$$q_{(n+1)} := \varpi q_{(n+1)} + (1-\varpi)q_{(n)}$$

where the operator := means updating the value of a variable. Setting a learning rate not too close to 1 is very important to guarantee the stability of the whole algorithm.

4. Stop when the absolute error is smaller than a threshold $\epsilon$:

$$\int\int \left|q_{(n+1)}(w,B) - q_{(n)}(w,B)\right| dwdB < \epsilon. \tag{A40}$$

The core of the algorithm is step 2, where the capital value function is updated. Solving the fixed point problem in (A37) requires knowledge of the global property for $q(w,B)$, for which I use the last round price function. This approach is valid because eventually, the consecutive rounds of iteration have very similar price functions and thus converge to the solution.

Below we provide the details for steps 1 and 2 in the above algorithm.

*Step 1: Initialization*

The whole algorithm needs an initialization of the function $q(w,B)$. I will initialize the algorithm by solving a simple version of the model without any bank run, i.e. $\lambda = 0$. Then the first order conditions of $x^K$ and $y^K$ imply

$$\mu^R + \frac{\bar{A}}{q} - r^d = (\sigma^K + \sigma^q)^2 x^K$$

$$\mu^R + \frac{\underline{A}}{q} - r^d = (\sigma^K + \sigma^q)^2 y^K$$

we get

$$\sigma^K + \sigma^q = \sqrt{\frac{\bar{A} - \underline{A}}{q(x^K - y^K)}}$$

Then from the volatility equation system, we can solve $q_w(w)$ as

$$q_w = \frac{\sigma^q}{w(1-w)(x^K - y^K)(\sigma^K + \sigma^q)} \tag{A41}$$

The boundary condition is $q(0) = \underline{q}$, which is solved from

$$\rho\underline{q} + i(\underline{q}) + g = \underline{A}$$

Clearly, the above equation system for $q(w,B)$ does not depend on $B$, and thus we can simply solve an ODE for the function $q(w,B) = q(w)$.

*Step 2: Update the Value function $q$*

At round $n$, for each pair of $(w, B)$, implement the following:

1. Start with last round price function $q_{(n)}(\cdot)$, solve for the current round $x^K$, $y^K$, and volatilities via equations (A16), (A21), (A19), and the definition of $\psi$ in (17). Then the volatilities, $\sigma^q$, $\sigma^h$, and $\sigma^b$, can be derived from equation (A28).
2. Next, solve $x^B$ via the market clearing conditions (A19). Solve $x^K$, $x^D$, and $\kappa^q$ together for equations (A23), (A24), (A26), (A27), and (A37).
3. With the updated $x^K$, we can solve for $\psi$ via (A20), where $q$ uses the last-round value $q_{(n)}$.
4. With the updated $\psi$, we solve for the next-round capital value through the consumption good clearing in (A21),

$$\psi \bar{A} + (1 - \psi)\underline{A} = \rho q_{(n+1)} + \phi(\mu^K(q_{(n+1)})) + g$$

Thus, after the above steps, we obtain $q_{(n+1)}$.

***Other Rates***

After solving for the capital value function $q$, we can follow the same algorithm to obtain the portfolio choices $x^K$, $x^D$, $x^B$, and the jumps in the capital value $\kappa^q$. Next, we need to calculate other rates, including $r^B$, $r^D$, and $r^{ND}$.

1. First solve for the spreads, $\mu^R - r^{ND}$, $r^{ND} - r^B$,$r^{ND} - r^D$, via the first-order conditions in (A23), (A25), and (A26).
2. Solve the drifts $\mu^b$, $\mu^h$, and $\mu^w$, through equations (A30), (A31), (A32). Then solve for $\mu^q$ via equation (A29). With $\mu^q$, we can solve for $\mu^R$ via its definition in (29).
3. With $\mu^R$ solved, we then obtain the level of rates, $r^B$, $r^D$, $r^{ND}$.

## *B.3. Calibration Moments*

Calibrated model parameters are shown in Table A1, and estimated parameters together with moment values are shown in Table A2.

Table A1: Calibrated Model Parameters

| | Parameters | Choice | Data Moment |
|---|---|---|---|
| $\lambda$ | Crises arrival rate | 4% | Frequency of historical financial crises |
| $\alpha_0$ | Market illiquidity discount | 21% | Loan haircut in the secondary market during the global financial crisis |
| $\beta$ | Fraction of runnable funding | 42 % | Runnable funding of banks from flow of funds, 1970 to 2016 |
| $\delta$ | Depreciation rate | 10% | Depreciation rate in the literature |
| $\rho$ | Time discount rate | 4% | Discount rate in the literature |
| $\chi$ | Investment adjustment cost | 3 | Adjustment cost in literature |
| $\varepsilon$ | Shareholder bankruptcy leftover | $10^{-3}$ | Small $\varepsilon$ |
| $\theta$ | Crisis shock size | $10^{-5}$ | Small $\theta$ |

Table A2: Moments and Model Estimates

**Panel A: Moments**

| Moment | Data | Model |
|---|---|---|
| Average of public liquidity/GDP | 38% | 38% |
| Minimum of public liquidity/GDP | 18% | 18% |
| Jump of public liquidity/GDP in crises | 8% | 8% |
| Average output change in a crisis | -8.0% | -7.6% |
| Average productivity | 0.14 | 0.14 |
| Average liquidity premium | 0.32% | 0.38% |
| Average deposit spread | 1.5% | 1.9% |
| Average insured deposits/total bank liability | 40% | 41% |
| Volatility of real GDP growth | 2.1% | 2.4% |
| Extra household growth due to deposits | 0 | 0 |
| Bank equity ratio | 10% | 12% |

**Panel B: Estimated Parameter Values**

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| Banker productivity $\bar{A}$ | 0.15 | Household productivity $\underline{A}$ | 0.12 |
| Capital growth volatility $\sigma^K$ | 3% | Banker→household transition $\eta$ | 0.2 |
| Household deposits valuation $\beta_D$ | 0.075 | Household deposits valuation $\underline{y}^D$ | 0.15 |
| Persistence of liquidity supply, $\theta_B$ | 0.018 | Crisis-expansion of liquidity $\kappa^B$ | 0.015 |
| Basic liquidity supply, $\bar{B}$ | 0.027 | Government spending $\bar{g}$ | 0.046 |
| Illiquidity parameter $\pi$ | 20% | | |

### *B.4. Additional Quantitative Evaluations of the Model*

In Table A3, I show additional untargeted moments and contrast them with the data. I simulate the model for 10000 years at monthly frequency and measure data moments on a sample that starts from 1920 (for most variables, this is the longest horizon of data I can obtain).

As shown in the panel A of Table A3, the model-implied volatilities are similar to the data counterparts for output growth, bank equity growth, consumption growth, investment growth, and the deposit spread. However, there are two major differences: First, the model-implied liquidity premium is not volatile enough compared to the data counterpart; Second, the model-implied risk-free rate is not volatile enough. These results are due to similar reasons explained in the previous exercise. We also note that although the underlying capital shock volatility is smaller than that of the previous exercise in Table 3, asset price volatilities are actually higher. The main reason is that the capital ratio data in the previous exercise is larger than the stationary state of the model, and thus the state vector is in the region with less volatile asset prices.

In panel B of Table A3, I report correlations among major quantities and prices. These correlations are of the same sign, and mostly of similar magnitudes. Nevertheless, the correlation between changes of deposit spread and output growth is much larger in the data than in the model. Deposit spread data for the long horizon are obtained from projections on federal funds rate, as in Krishnamurthy and Li (2023), and thus, highly correlated with output dynamics due to the endogenous reaction of monetary policy. However, there is no such link in the model and therefore the model-implied correlation is tiny.

Table A3: Additional Untargeted Model Moments

| Moments (all units are %) | Model | Data |
|---|---|---|
| Panel A: volatilities | | |
| vol(liquidity premium) | 0.17 | 0.30 |
| vol(output growth) | 2.10 | 5.01 |
| vol(bank equity growth) | 30.56 | 26.06 |
| vol(consumption growth) | 2.75 | 1.94 |
| vol(investment growth) | 4.28 | 6.80 |
| vol(risk free rate) | 1.13 | 3.28 |
| vol(deposit spread) | 1.32 | 1.12 |
| Panel B: correlations | | |
| corr(consumption growth, output growth) | 0.88 | 0.85 |
| corr(investment growth, output growth) | 0.77 | 0.70 |
| corr(bank equity growth, output growth) | 0.14 | 0.31 |
| corr(diff(liquidity premium), output growth) | -0.07 | -0.08 |
| corr(diff(risk free rate), output growth) | 0.10 | 0.19 |
| corr(diff(deposit spread), output growth) | 0.01 | 0.19 |

*Notes*: this table shows model moments from a model simulation of 10000 years at monthly frequency. All growth variables are calculated as yearly growth, and all differences are taken as yearly differences. Data are from 1920 to 2016, except for bank equity growth, which is the same as He and Krishnamurthy (2019) and starts from 1973. The risk free rate is the federal funds rate. The deposit spread is from Krishnamurthy and Li (2023).

# Appendix C. Additional Empirics

In this section, I will present additional VAR analysis, regressions evidence about the liquidity premium, and discuss alternative measures of the liquidity premium.

## *C.1. Robustness of the VAR Analysis*

The VAR setup is

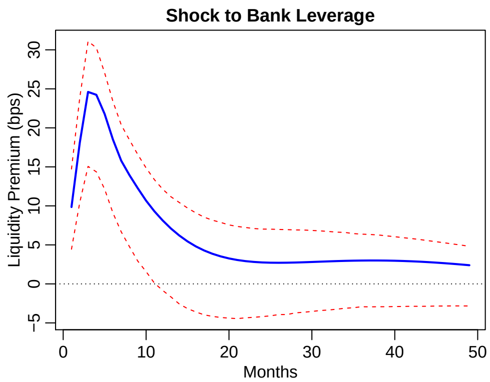
$$X_t = A_1 X_{t-1} + \cdots + A_p X_{t-p} + u_t \tag{A42}$$

$$\begin{aligned} X_t = [&\Delta \text{GDP}_t, \Delta \text{C\&I loans}_t, \Delta \text{investment}_t, (\text{public liquidity/GDP})_t, \\ &\text{inflation}_t, \text{federal funds rate}_t, \text{stock market return}_t, \\ &\text{bank leverage}_t, \text{excess bond premium}_t, \text{liquidity premium}_t] \end{aligned} \tag{A43}$$

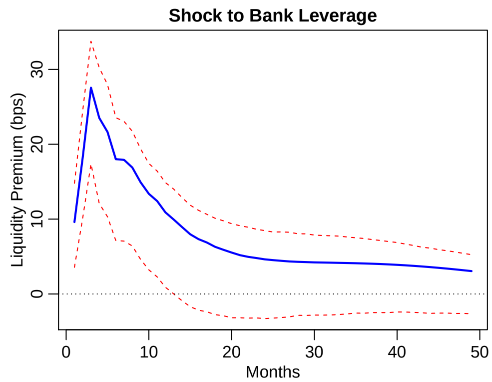
In the following, I will present more robustness checks on the main result – the liquidity premium significantly responds to a shock of bank leverage. As shown in Figure A1, the impulse response remains significant and positive after a variety of changes, including setting lag $p = 3$ and $p = 4$, excluding recession and crisis episodes (with $p = 2$), and putting bank leverage at the beginning of the variable list (with $p = 1$ to avoid duplication of Figure 1b).

Next, I show the impulse response results for other variables in Figure A2. Results are broadly sensible, with the liquidity premium positively reacting to a C&I loan growth shock, which proxies firms' demand for liquidity. Furthermore, the impulse response to a public liquidity/GDP shock is negative, consistent with the literature. The error bands are wider than other shocks, since in the data, we only have measures on quarterly government debt volume, while the analysis is done at a monthly frequency (government debt volume is interpolated within each quarter). A longer period analysis at a lower frequency generally reveals a stronger relationship, as in Krishnamurthy and Vissing-Jorgensen (2012). Finally, we find that initially, the liquidity premium positively responds to a credit spread shock, but then the response reverts and becomes slightly negative, and eventually converges towards zero.

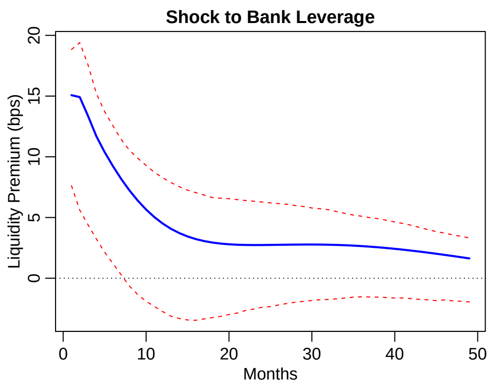
Since in the model, intermediaries charge a higher risk premium when leverage is high, we also expect that the excess bond premium positively responds to a positive leverage shock. Indeed, as shown in Figure A3, the initial response of excess bond premium to a bank leverage shock is significantly positive, although followed by a slight reversal that is not statistically different from zero.



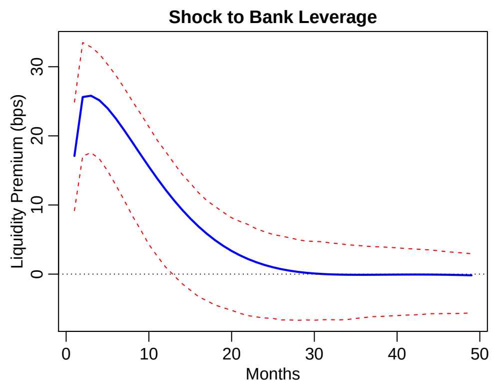

(c) lag $p = 3$




(d) lag $p = 4$




(e) Excluding Recession/Crises




(f) Ranking Bank Leverage at Front

Figure A1. Various Robustness Checks on the Impulse Responses of Liquidity Premium to Bank Leverage. The dashed red lines illustrate the 90% confidence interval.

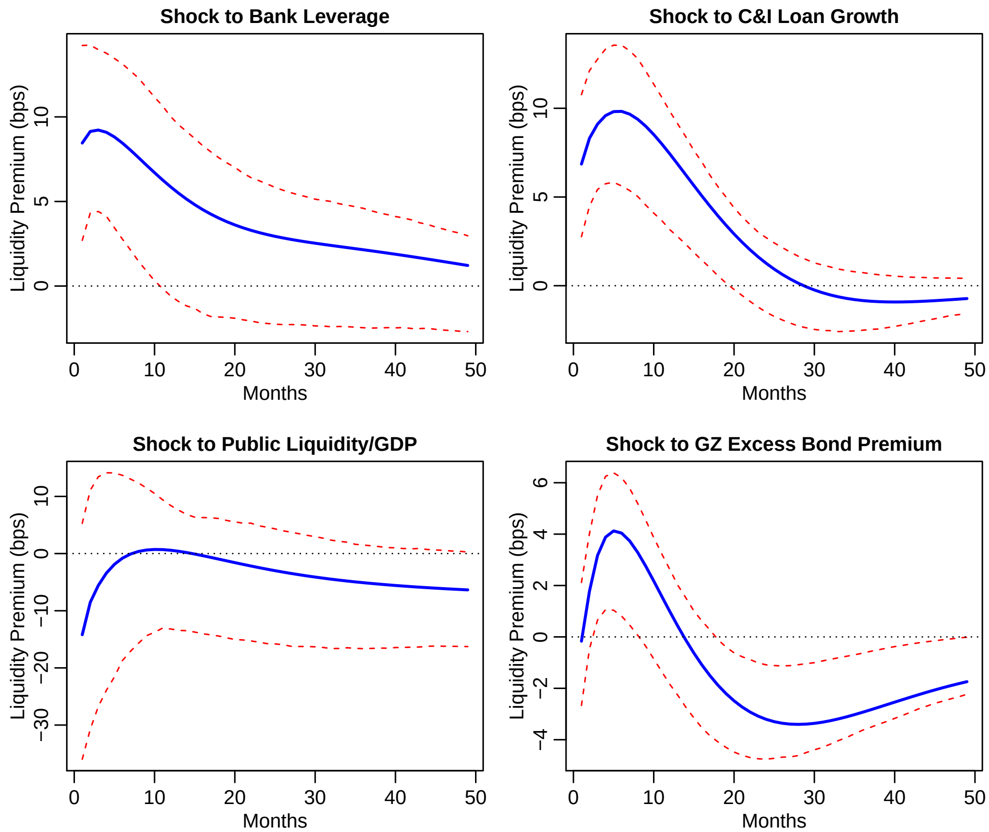



Figure A2. Impulses Responses of the Liquidity Premium to Other Shocks. This figure shows the impulse response of the liquidity premium to a one standard deviation shock to other variables, including C&I loan growth that proxies firm liquidity demand, public liquidity supply/GDP, and the shock to GZ excess bond premium. The dashed red lines illustrate the 90% confidence interval.

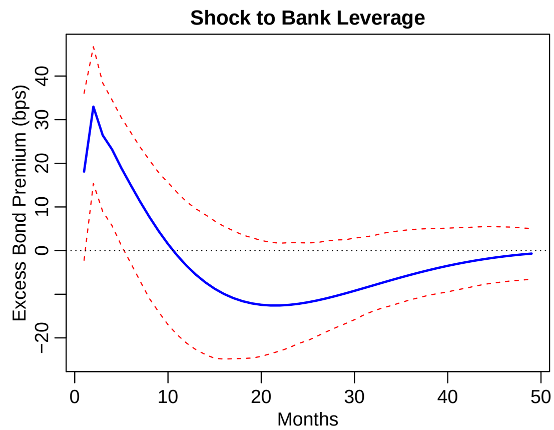



Figure A3. Impulse response of Excess Bond Premium to Bank Leverage. This figure shows the impulse response of excess bond premium as in Gilchrist and Zakrajšek (2012) to a one standard deviation shock of bank leverage.

## *C.2. Narrative Restrictions*

In this subsection, I follow the methodology of Antolín-Díaz and Rubio-Ramírez (2018) to implement the narrative restrictions on VAR analysis. The idea is that there are episodes of events when banking disruptions are the key and dominate the response of the liquidity premium to other variables. By imposing such economic knowledge onto the VAR system, we are better able to identify how banking-sector shocks affect the pricing of liquidity and other dynamics of the economy. Since the key innovation of Antolín-Díaz and Rubio-Ramírez (2018) is to introduce narrative restrictions, not sign restrictions, I focus on the implementation of narrative restrictions.

The starting point of the analysis is a traditional VAR with rank restrictions, and then the algorithm imposes narrative restrictions with a Bayesian approach. I use the same baseline VAR as in Section 2.1. Then, I pick the same events as in Section C.3 and impose the restriction that during those events (since the VAR analysis is at monthly frequency, the restriction is on the event month), the absolute value of the liquidity premium response to the bank leverage shock dominates the absolute value of response to any other variable in the VAR. For identification purpose, I also include a sign restriction that the response of leverage to leverage shock is positive.

The resulting impulse responses are illustrated in Figure A4, with confidence interval of plus and minus one standard deviation. We find that the liquidity premium positively responds to bank leverage shocks and the effect persists for about a year. Consistent with the theory, the excess bond premium also positively responds to a positive bank leverage shock. Next, a positive leverage shock negatively affects bank lending, and therefore, reduces

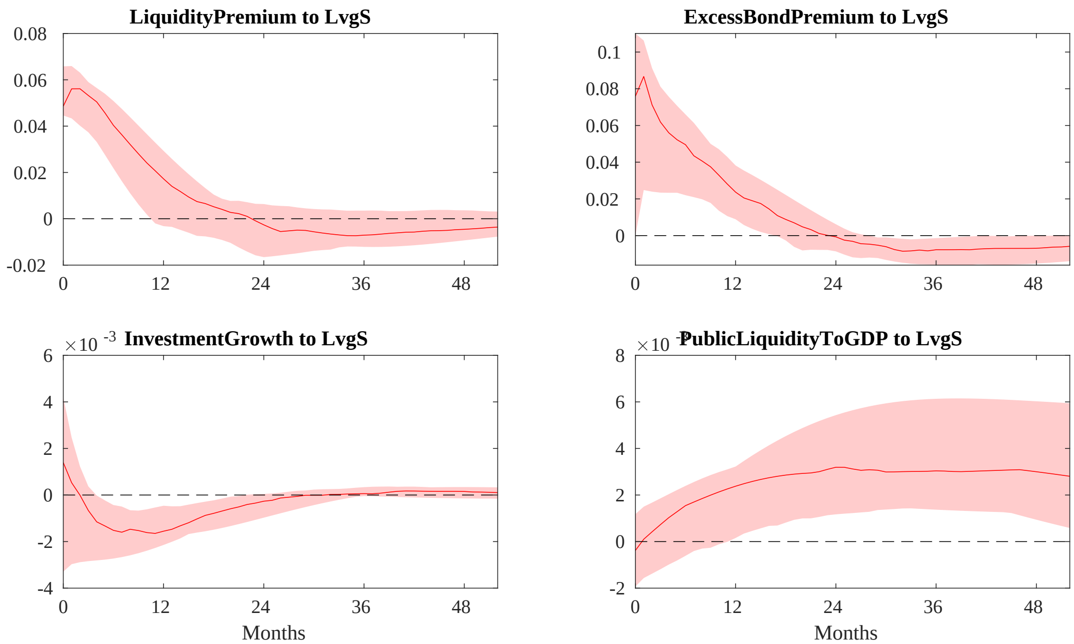



Figure A4. Impulse Responses with Narrative Restrictions on the VAR

investment growth. Finally, the government will react to those events, leading to a higher public liquidity/GDP ratio afterward.

## *C.3. Event Studies*

Next, I study events that are closely linked to bank shocks and show the strong comovements between bank leverage and the liquidity premium. To get better identification of asset price changes around those events, I use daily data of both bank leverage (inverse of bank capital ratio from He, Kelly and Manela (2017)) and the liquidity premium from Nagel (2016).[17]

I use the following events:

- September 11, 2001: the terrorist attack that destroyed the World Trade Center and caused damage to the financial sector (McAndrews and Potter, 2002).
- September 15, 2008: the bankruptcy of Lehman Brothers during Global Financial Crisis.[18]

[17] The repo-based measure from Nagel (2016) has more sensitive high-frequency variation than the Refcorp-based measure in Longstaff (2004), so I use only the measure from Nagel (2016) for this high-frequency exercise.

[18] See Brunnermeier (2009) for more detailed analysis on Lehman bankruptcy and why it is such a major liquidity shock to the banking sector..

- May 7, 2009: release of bank stress testing results that reveal bank health during Global Financial Crisis.[19]
- November 1, 2009: the bankruptcy of CIT Group.[20]
- May 10, 2010: Eurozone leaders resolved in Brussels to take drastic action against the debt crisis.[21]
- July 26, 2012: the "whatever it takes" speech by ECB President Mario Draghi.

Results are shown in Figure A5. I normalize both bank leverage and the liquidity premium for comparison. We find that in all of these events, bank leverage and the liquidity premium strongly comove. To the extent that these events represent shocks to banks, results provide supportive evidence on bank demand of liquidity driving the liquidity premium.

These events can also be integrated into the VAR analysis in Section 2.1, using the narrative restriction approach in Antolín-Díaz and Rubio-Ramírez (2018). Since this approach is more involved, I illustrate the results in Appendix C.2. With narrative restrictions, the liquidity premium response to bank leverage shocks is significant and positive, consistent with previous results.

## *C.4. Regressions on the Liquidity Premium using a Long Sample*

In the analysis of Table 2, we find that public liquidity has a small explanatory power on the liquidity premium. The reason is that the public liquidity supply is slow-moving, and there are not enough variations during that period. Once we extend the data sample to a longer horizon as in Krishnamurthy and Vissing-Jorgensen (2012), there is a significant negative relationship between public liquidity supply and the liquidity premium. As shown in Table A4 below, public liquidity supply can explain 12% of time-series variations in monthly liquidity premium, for the data sample from 1929 to 2016. Despite the difference in $R^2$, the coefficient on public liquidity/GDP is very similar to columns 1-3 of Table 2.

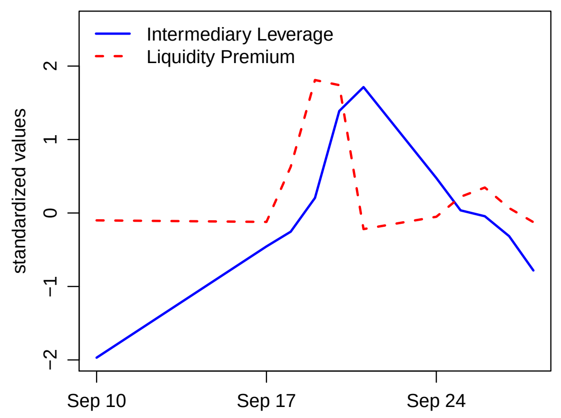
## *C.5. Alternative Measures of the Liquidity Premium*

In the paper, I have used the principal component of GC Repo 3 month term loan spread with respect to treasury 3 months, as well as the Refcorp – Treasury spreads as the liquidity premium measure from 1991 to 2016. This is an ideal measure for the liquidity premium, because it has no credit risk in all components and is more robust by extracting

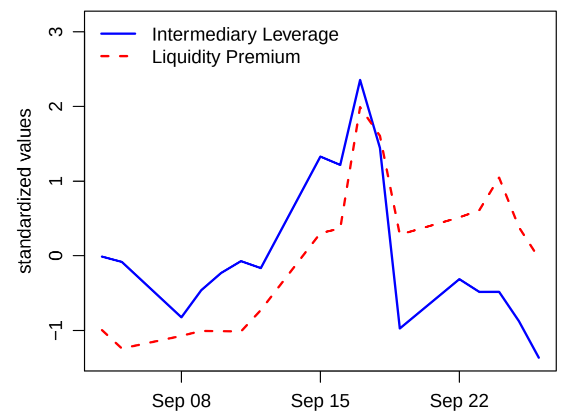
[19] Link to the Federal Reserve announcement: `https://www.federalreserve.gov/newsevents/pressreleases/bcreg20090507a.htm`

[20] Refer to Helwege and Zhang (2016) for further analysis on the significance of financial firms' bankruptcies.

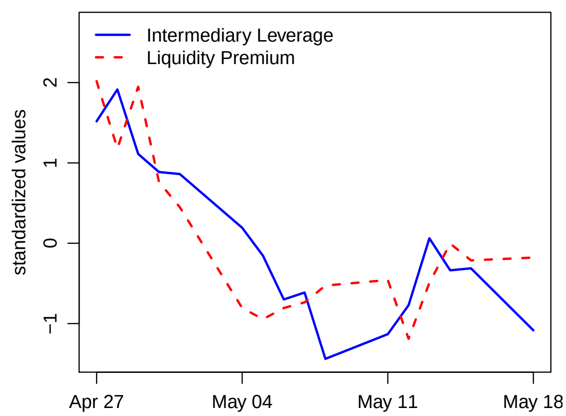
[21] See a list of European debt crisis events in Table 2 of Stracca (2013). I only select a subset of those events that are most pronounced and mostly related to the banking sector.



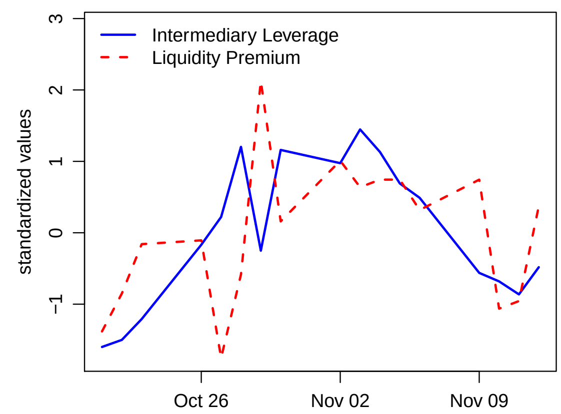

(a) 9-11 Terrorist Attack



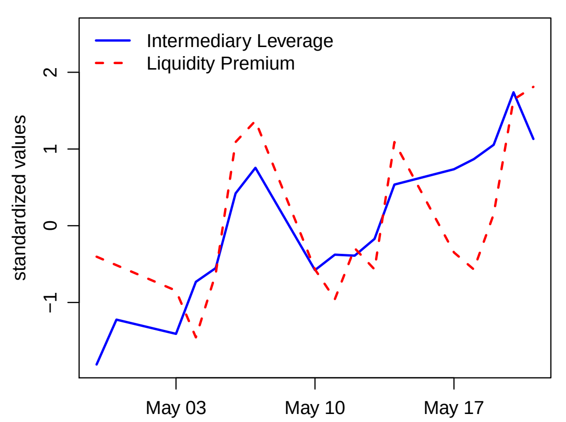

(b) Lehman Brothers bankruptcy



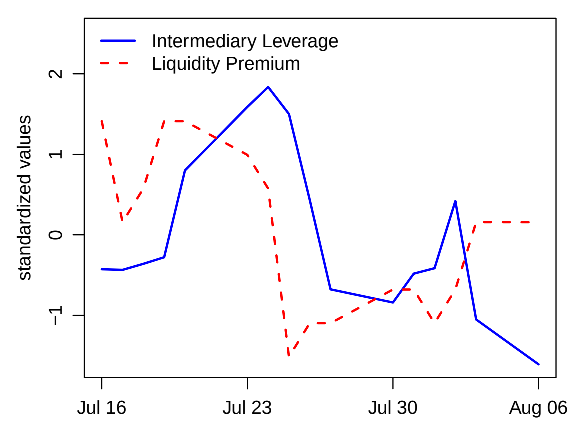

(c) Release of Bank Stress Testing Results




(d) Bankruptcy of CIT Group




(e) Eurozone Meeting on Debt Crisis




(f) "Whatever it takes" by Draghi

Figure A5. **Event Studies**. This figure plots the path of intermediary leverage and the liquidity premium around six events of banking-sector disruptions. Both variables are standardized as zero-mean and unit-volatility for comparison.

Table A4: Empirical Relationships between the Liquidity Premium and Public Liquidity Supply at a Longer Horizon

| | Liquidity Premium |
|---|---|
| public liquidity/GDP | $-0.51^{**}$ |
| | (0.20) |
| Constant | $0.49^{***}$ |
| | (0.10) |
| Observations | 1,056 |
| $R^2$ | 0.12 |

*Notes*: Public liquidity is defined as the total government bonds held by the domestic private sector plus central bank reserves. Data are from 1929 to 2016.

the common variations. However, before 1991, we do not have such a measure, and the banker's acceptance might have a small credit risk component. Then it is necessary to check if the main results in the paper are robust to alternative measures to the liquidity premium before 1991. Since the 2008 counterfactual analyses are only based on the Repo spread data, none of the results in section 6 will be affected.

One alternative measure is the fed funds rate – treasury 3-month spread. It is not an ideal measure as well, because the uninsured interbank borrowing and lending also have credit risks. Furthermore, the fed funds rate is an overnight rate, which makes the measure subject to maturity mismatch. To fix this issue, I can also use another measure: the compounded 3-month fed funds rate – treasury 3-month spread. The compounded 3-month rate is an average of fed funds rate in the next 3 months. Absent from monetary policy shocks, this should be a good measure for the expectation of interest rate in the coming 3 months. However, since the fed funds rate has unexpected shocks, this measure might include too much noise and reduce the power of explanation.

In Figure A6, I plot the three spreads based on three different measures, including the 3-month baker acceptance, fed funds rate, and compounded fed funds rate for 3 months. We find that the three measures are very close to each other from 1970 to 2000. Before 1970, the spreads based on the fed funds rate and the compounded fed funds rate are still similar, but the compounded fed funds rate–treasury spread has more fluctuations, indicating unexpected shocks. However, compared to the banker acceptance spread, the spreads based on the fed funds rate are much higher before 1950. Three reasons may lead to this phenomenon. First,

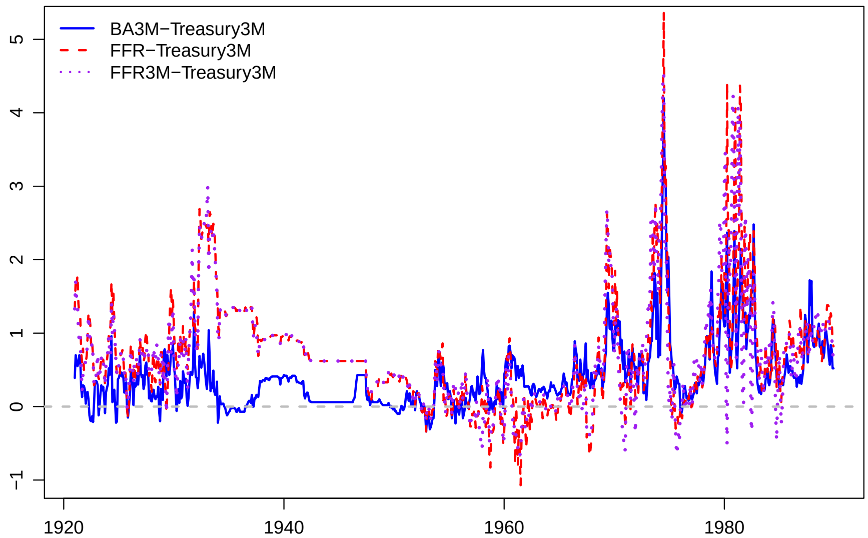



Figure A6. **Alternative Measures of the Liquidity Premium**. This figure illustrates the time series of several different measures of the liquidity premium. BA3M refers to the yield of the three-month banker acceptance. Treasury3M refers to the yield of three-month Treasurys. FFR is the federal funds rate, while FFR3M is the compounded 3-month federal funds rate.

since the banker's acceptance was widely used for international trade and accessible by a large group of investors, it is more liquid than the fed funds. Second, banker's acceptance was backed and directly purchased by the federal reserve for the majority of the period before 1977, making it safer than the interbank borrowing. Third, the payment from a banker's acceptance is double-backed by both the bank and the underlying firm. Consequently, the current measure of the liquidity premium, the banker's acceptance – treasury 3-month spread, is the best among these proposed alternatives.