Liquidity Premium and the Substitution between Money and Treasuries

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Substitution between Money and Treasuries

- Money and treasuries both provide liquidity services.

- Question: Are money and treasuries substitutable? If so, what is the level of substitution?

- Information revealed by the treasury liquidity premium.
  - Treasuries have liquidity premium for their liquidity service \( \sim 0.2\% \) of GDP, about $40B in 2016.
  - The liquidity premium declines with treasury supply (Krishnamurthy and Vissing-Jorgensen 2012).
  - Treasury supply is insignificant once we include the interest rate: Money and treasuries are perfect substitutes (Nagel 2016)
Liquidity premium and the Level of Substitution

- Intuitions from Nagel 2016.
  - If money $\sim$ treasuries, then
    \[
    \text{liquidity premium} \leftarrow \text{fed funds rate}
    \]
  - If money $\sim$ treasuries, then
    \[
    \text{liquidity premium} \leftarrow \text{supply of treasuries}
    \]
  - Horse race:
    \[
    \text{liquidity premium} \leftarrow \text{fed funds rate} + \text{supply of treasuries}
    \]
Preview of Results

- My finding: Money (bank deposits) and treasuries are not close substitutes. Use general CES aggregator with substitution $\rho$

  $$Q_t = \left( (1 - \lambda_t) \left( \frac{D_t}{P_t} \right)^\rho + \lambda_t \left( \frac{B_t}{P_t} \right)^\rho \right)^\frac{1}{\rho}$$

  GMM result: $\rho = 0.277 \ll 1$.

- Key improvement: structural estimation with more information.

  liquidity premium$_t = f(i_t) \cdot g(B_t/D_t)$

  1. More information: include quantity and rates of deposits.
  2. The form of interaction.
Importance of Understanding the Level of Substitution

- Monetary policy and monetary models.
  - Composition and substitution of money and treasuries $\Rightarrow$ Effectiveness of open market operations / Impact of change in treasuries on interest rate.
  - Current building blocks of monetary models do not have treasuries in the liquidity.

- Treasury liquidity premium.
  - A good indicator of financial distress (Longstaff 2002).
  - Connected to risk premia and risk-taking (Drechsler, Savov, Schnabl 2018; Li 2018).
  - Closely related to exchange rates (Jiang, Krishnamurthy, and Lustig 2018).
Outline

1. The Model
2. GMM Estimations on the Level of Substitution
3. Evaluations of Different Methods
4. Summary
Setup

- Representative household has utility function

\[ E_0[\sum_{t=1}^{\infty} \beta^t (u(C_t) + \alpha \cdot v(Q_t))] \]

\[ Q_t = \left( (1 - \lambda_t) \left( \frac{D_t}{P_t} \right)^\rho + \lambda_t \left( \frac{B_t}{P_t} \right)^\rho \right)^{\frac{1}{\rho}} \]

- Budget constraint

\[
\begin{align*}
D_{t-1}(1 + i_{t-1}^d) + B_{t-1}(1 + i_{t-1}^b) + A_{t-1}(1 + i_{t-1}) + I_t = \\
\underbrace{P_t C_t} \quad \underbrace{+ D_t} \quad \underbrace{+ B_t} \quad \underbrace{+ A_t} \quad \underbrace{+ T_t} \\
\text{consumption} \quad \text{investment} \quad \text{transfer}
\end{align*}
\]
First Order Conditions

- Household FOC on bonds:
  \[ u'(C_t) \frac{i_t - i^b_t}{1 + i_t} = \alpha' \left( Q_t \right) Q_t^{1-\rho} \rho \lambda_t \left( \frac{B_t}{P_t} \right)^{\rho-1} \]

- Household FOC on deposits:
  \[ u'(C_t) \frac{i_t - i^d_t}{1 + i_t} = \alpha' \left( Q_t \right) Q_t^{1-\rho} \rho (1 - \lambda_t) \left( \frac{D_t}{P_t} \right)^{\rho-1} \]

- Divide on both sides:
  \[ \frac{i_t - i^b_t}{i_t - i^d_t} = \tilde{\lambda}_t \left( \frac{B_t}{D_t} \right)^{\rho-1} \]

  with \( \tilde{\lambda}_t = \lambda_t / (1 - \lambda_t) \) the relative demand.
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Measurements

- What to include in “money”? Substitution w.r.t. treasuries available.
  - Currency: held by investors mostly outside U.S., without substitution.
  - Time deposits, MMF: households hold treasuries through them.

- Definition of deposits:
  \[
  d_t = 2(\delta(d_{\text{checking}},t)^\kappa + (1 - \delta)(d_{\text{saving}},t)^\kappa)^{\frac{1}{\kappa}}
  \]

- Estimation of \( \kappa \) and \( \delta \).
  \[
  \frac{i_t - i_{\text{checking}},t}{i_t - i_{\text{saving}},t} = \frac{\delta}{1 - \delta} \left( \frac{d_{\text{checking}},t}{d_{\text{saving},t}} \right)^{\kappa - 1}
  \]

- Data: monthly deposit rates (1986-2016) from call report; monthly deposit volume (1959-2016) from FRED.
- Results: \( \kappa = 1 \) and \( \delta = 2/3 \).
Measurements

- Equation for estimation:

\[
\begin{align*}
\text{liquidity premium, } lp_t & = \tilde{\lambda}_t \cdot \frac{B_t}{D_t}^{\rho - 1} \cdot (i_t - i^d_t) \\
\end{align*}
\]

- Measurements:
  - Liquidity premium \( lp_t \). For 1991-2016: 3 month Repo collateralized - 3 month treasuries. For 1920-1990, 3 month banker acceptance - ...
  - \( \tilde{\lambda}_t = \beta_\lambda \cdot \text{VIX}_t \). Reconstructed from stock returns, 1920-2016.
  - \( B_t \): treasuries - foreign and intra-government holding, 1920-2016.
  - Deposit spread:

\[
\begin{align*}
 i_t - i^d_t & = ( (i_t - i_{\text{checking},t})d_{\text{checking},t} + d_{\text{saving},t}(i_t - i_{\text{saving},t}) ) / d_t \\
 i_t - i^d_t & \approx \delta_i_t. \text{ About 90% variations captured. FFR data 1920-2016.}
\end{align*}
\]
Estimation Methods

- Denote the residual of model prediction as

\[ \varepsilon_t = lp_t - \beta_\lambda VIX_t \left( \frac{B_t}{D_t} \right)^{\rho - 1} \cdot (\delta i_t) \]

- Method: GMM with two types of moments. The first type include FOCs of

\[ \min_{\rho, \beta_\lambda} \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t^2 \]  

\[ \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \cdot \delta i_t \cdot VIX_t \left( \frac{B_t}{D_t} \right)^{\rho - 1} = 0 \]  

\[ \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \cdot \delta i_t \cdot VIX_t \left( \frac{B_t}{D_t} \right)^{\rho - 2} = 0 \]  

- Second type is about residual restrictions,

\[ \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t = 0 \]  

(residual restriction)
Two-step GMM with Newey-West standard errors of 12 lags.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.277</td>
<td>(0.157)</td>
</tr>
<tr>
<td>$\beta_\lambda$</td>
<td>0.010***</td>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

Observations: 696 (1959-2016, monthly)

p-value of J-test: 0.818

Note: * $p<0.1$; ** $p<0.05$; *** $p<0.01$
Estimation Results

- Two-step GMM with Newey-West standard errors of 12 lags.

<table>
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<th></th>
<th>Value</th>
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<tr>
<td>$\rho$</td>
<td>0.277* (0.157)</td>
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<td>$\beta_\lambda$</td>
<td>0.010*** (0.0005)</td>
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Observations: 696 (1959-2016, monthly)
P-value of J-test: 0.818

Note: *p<0.1; **p<0.05; ***p<0.01

- Wald test of $\rho = 1$ has a p-value $= 4.4 \times 10^{-6}$ and strongly rejects perfect substitution.
Probability of Wrong GMM Estimations

- What is the probability of getting $\hat{\rho} \leq 0.277$ given $\rho = 1$?
- Assume the underlying model is the same as the regressions in Nagel (2016),

$$lp_t = \alpha_0 + \alpha_1 i_t + \alpha_2 \text{VIX}_t + \varepsilon_t$$

Then use stationary bootstrap on the residual and apply GMM estimations.

![Density plot showing GMM estimations of $\rho_{-1} = \rho - 1$ with probability of $\hat{\rho} < 0.277$ and $\hat{\rho} < -0.723$ calculated as $0.04\%$.](image)
Model Fit

- Model predicted liquidity premium versus data.
Decompose Different Components

- Model predictions with only variations in FFR, versus data.
Decompose Different Components

- Model predictions with variations in FFR and VIX, versus data.
Decompose Different Components

- Full model predictions, versus data.
Changes of Substitution Over Time

- Before 1959 (back to 1934), we only have yearly deposits data from FDIC. But a longer history is needed for estimating the changes of substitution.

<table>
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<tr>
<th>Parameters</th>
<th>Estimated Value (Volatility)</th>
</tr>
</thead>
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<tr>
<td>$\rho$</td>
<td>0.019 (0.381)</td>
</tr>
<tr>
<td>$\beta\lambda$</td>
<td>0.019*** (0.004)</td>
</tr>
<tr>
<td>Observations (yearly)</td>
<td>36</td>
</tr>
<tr>
<td>p-value of J-tests</td>
<td>0.847</td>
</tr>
</tbody>
</table>

Note: * $p<0.1$; ** $p<0.05$; *** $p<0.01$
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Different Estimation Methods

- Underlying model:

\[
lp_t = \beta \lambda \text{VIX}_t \cdot \left( \frac{B_t}{D_t} \right)^{\rho - 1} \cdot (\delta i_t) + \varepsilon_t
\]

- Compare three different methods:

  1. Linear regressions in Nagel (2016):

    \[
    lp_t \leftarrow \text{VIX}_t + \log\left(\frac{B_t}{GDP_t}\right) + i_t
    \]

  2. Linear regression using Treasuries/Deposit:

    \[
    lp_t \leftarrow \text{VIX}_t + \log\left(\frac{B_t}{D_t}\right) + i_t
    \]

  3. GMM regressions. Estimate \( \rho_{-1} = \rho - 1 \).
Evaluations of Model Estimation Methods

- My estimation: $\rho = 0.277$. Conclusion in Nagel 2016: $\rho \approx 1$. Differences:
  1. More info: I include $D_t$.
  2. GMM accounts for the interaction between $i_t$ and $B_t/D_t$.

- Approach 1: Stationary bootstrap (closer to the data)
  - Extract model residual $\varepsilon_t$ under estimated parameters.
  - Bootstrap $\varepsilon_t$ to generate new monthly data $lp_t$.

- Approach 2: Pure simulation (closer to theoretical analysis)
  - Simulate all the elements of the model, subject to: $B_t$ and $D_t$ change at lower frequencies than $i_t$, and have smaller range of variations.
  - Generate $\varepsilon_t$ independent from others.
Approach 1: Stationary Bootstrap

- Coefficient estimations in different rounds of bootstraps.

![Graph showing coefficient estimations for different methods]
Estimated t-stats Using Bootstrap

- Histogram on t-stats of liquidity supply in linear regressions and $\rho - 1$ in GMM regressions.
Approach 2: Pure Simulation

Q: What is the fundamental problem of using linear specifications for estimating the model

\[ lp_t = \beta \lambda VIX_t \left( \frac{B_t}{D_t} \right)^{\rho - 1} (\delta i_t) + \varepsilon_t \]

Pure simulation with the following key features:

- \( B_t/GDP_t \) and \( D_t/GDP_t \) are at yearly frequency, while all others are at monthly frequency.

- \( i_t \) has much larger range than \( B_t/GDP_t \) and \( D_t/GDP_t \).
Distribution of t-stats under $\sigma = 0.01$

- Histogram on t-stats of liquidity supply in linear regressions and $\rho - 1$ in GMM regressions.
Distribution of t-stats under $\sigma = 0.006$

- Histogram on t-stats of liquidity supply in linear regressions and $\rho - 1$ in GMM regressions.
Distribution of \( t \)-stats under \( \sigma = 0.002 \)

- Histogram on \( t \)-stats of liquidity supply in linear regressions and \( \rho - 1 \) in GMM regressions.

![Histogram](image-url)
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Summary

- Level of substitution between bonds and money from 1959-2016:
  \[ \hat{\rho} = 0.277 \]

  which increases from about 0 to 0.6 from 1934-1970 to 1980-2016.

- Implications
  - Treasury supply directly affects the liquidity premium, but also partially puts pressure on the nominal interest rate. The Fed adjusts accordingly to dampen the effect.
  - With technological progress, the boundary between bonds and money becomes blurry.
  - Liquidity premium of treasuries are affected by both treasury supply and FFR. \( \Rightarrow \) Exchange rate movements.

Simulation Details

\[ lp_t = \beta \lambda \text{VIX}_t \left( \frac{B_t}{D_t} \right)^{\rho-1} (\delta i_t) + \varepsilon_t \]

- Simulation details:
  - Treasuries/GDP and Deposits/GDP are simulated based on yearly variations, and generated by rescaling \( e^{x_t} / (1 + e^{x_t}) \), with \( x_t \) independent AR(1) processes of volatility 0.05 and persistence 0.99.
  - FFR is simulated monthly as rescaled \( e^{y_t} \), with \( y_t \) an AR(1) process of volatility 0.05 and persistence 0.99.
  - VIX is simulated monthly as \( e^{z_t} \), with \( z_t \) an AR(1) process of volatility 0.1 and persistence 0.99.
  - \( \varepsilon_t \) is a monthly AR(1) process with persistence 0.5, and standard errors \( \sigma \).